

Section 1.1 – Real Numbers

1, 2, 3, ...

Natural Numbers ($\mathbb{N}$)

..., -3, -2, -1, 0

Integers ($\mathbb{Z}$)

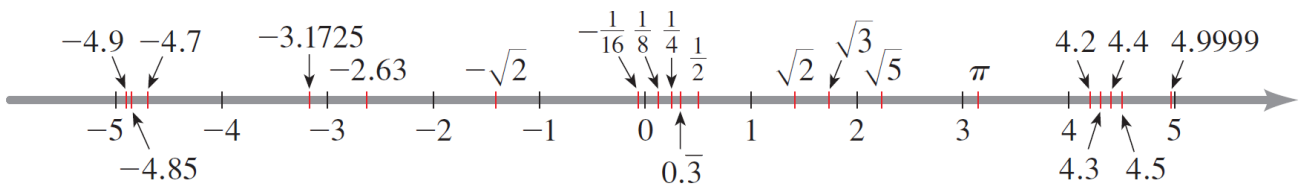
$\frac{1}{2}, -\frac{3}{7}, 46, 0.17, 0.6, 0.317$

Rational Numbers ($\mathbb{Q}$)

$\sqrt{2}, \sqrt{5}, \sqrt[3]{2}, \pi$

Irrational Numbers

Real Numbers ($\mathbb{R}$)



لاحظ

Rational Numbers ($\mathbb{Q}$) can be expressed in three formats:

- Fraction

$$\frac{1}{2}$$

$$-\frac{3}{7}$$

$$46 = \frac{46}{1}$$

- Finite decimal numbers

0.17

0.5

- Repeating decimal representation

$$0.\overline{6} = 0.66666 \dots$$

$$0.3\overline{17} = 0.3171717 \dots$$

Example 1Express the decimal $0.2\overline{8}$ as a fraction.**Solution**

$$x = 0.2\overline{8} = 0.28888\ldots$$

$$100x = 28.8888\ldots$$

$$10x = 2.8888\ldots$$

$$\begin{array}{r} 100x = 28.8888\ldots \\ - 10x = 2.8888\ldots \\ \hline 90x = 26 \end{array}$$

$$x = \frac{26}{90}$$

- Properties of real numbers خصائص الأعداد الحقيقية

Commutative

$$a + b = b + a$$

$$3 + 4 = 4 + 3$$

$$ab = ba$$

$$5 \cdot 6 = 6 \cdot 5$$

Associative

$$(a + b) + c = a + (b + c)$$

$$(2 + 3) + 7 = 2 + (3 + 7)$$

$$(ab)c = a(bc)$$

$$(4 \cdot 5) \cdot 3 = 4 \cdot (5 \cdot 3)$$

Distributive

$$a(b + c) = ab + ac$$

$$2 \cdot (6 + 4) = 2 \cdot 6 + 2 \cdot 4$$

$$(b + c)a = ab + ac$$

$$(6 + 4) \cdot 2 = 2 \cdot 6 + 2 \cdot 4$$

Example 2

Use properties of real numbers to write the expression without parentheses:

(a) $3(x + y)$

(b) $(3a)(b + c - 2d)$

Solution

$$(a) \quad 3(x + y) = 3x + 3y$$

$$(b) \quad (3a)(b + c - 2d) = 3ab + 3ac - 6ad$$

- The number 0 is called the **additive identity** المحاييد الجمعي because $a + 0 = a$ for any real number a .

- **Properties of negatives**

1. $(-1)a = -a$

2. $-(-a) = a$

3. $(-a)b = a(-b) = -(ab)$

4. $(-a)(-b) = ab$

5. $-(a + b) = -a - b$

6. $-(a - b) = b - a$

Example 3

Use properties of negatives to write the expression $-(x + y - z)$ without parentheses.

Solution

$$-x - y + z$$

- The number 1 is called the **multiplicative identity** المحاييد الضربي because $a \cdot 1 = a$ for any real number a .

- Division القسمة is the inverse of multiplication, hence:

$$a \div b = a \cdot \frac{1}{b}$$

$$= \frac{a}{b}$$

Numerator البسط

Denominator المقام

- Properties of fractions

$$1. \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$$

$$2. \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$$

$$3. \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$$

$$4. \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7 + 3 \cdot 5}{5 \cdot 7} = \frac{14+15}{35} = \frac{29}{35}$$

$$5. \frac{ac}{bc} = \frac{a}{b}$$

$$\frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$$

$$6. \text{ If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc$$

$$\frac{2}{3} = \frac{6}{9}, \text{ so } 2 \cdot 9 = 3 \cdot 6$$

Example 4

Perform the indicated operations.

(a) $\frac{3}{10} + \frac{4}{15}$

(b) $\frac{2}{3} (6 - \frac{3}{2})$

(c) $\frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}}$

Solution

$$(a) \frac{3}{10} + \frac{4}{15}$$

$$= \frac{3 \times 15 + 4 \times 10}{10 \times 15} = \frac{45 + 40}{150}$$

$$= \frac{85}{150}$$

$$(b) \frac{2}{3} (6 - \frac{3}{2}) = \frac{2}{3} \cdot 6 - \frac{2}{3} \cdot \frac{3}{2}$$

$$= \frac{12}{3} - \frac{6}{6} = 4 - 1 = 3$$

$$(c) \frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}} = (\frac{2}{5} + \frac{1}{2}) \div (\frac{1}{10} + \frac{3}{15})$$

$$= (\frac{2 \times 2 + 1 \times 5}{5 \times 2}) \div (\frac{1 \times 15 + 3 \times 10}{10 \times 15})$$

$$= (\frac{9}{10}) \div (\frac{45}{150})$$

$$= \frac{9}{10} \times \frac{150}{45} = \frac{1350}{450} = 3$$

- If a is a real number, then the **absolute value** مطلق of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

- **Properties of absolute values**

- | | |
|---|--|
| 1. $ a \geq 0$ | $ -3 = 3 \geq 0$ |
| 2. $ a = -a $ | $ 5 = -5 $ |
| 3. $ ab = a b $ | $ -3 \cdot 5 = -3 5 $ |
| 4. $\left \frac{a}{b}\right = \frac{ a }{ b }$ | $\left \frac{12}{-3}\right = \frac{ 12 }{ -3 }$ |
| 5. $ a + b \leq a + b $ | $ -3 + 5 \leq -3 + 5 $ |

Example 5

Evaluate each expression.

(a) $|-6| - |-4|$

(b) $\left|\frac{7-12}{12-7}\right|$

Solution

$$(a) |6 - 4| = |2| = 2$$

$$(b) \left| \frac{-5}{5} \right| = |-1| = 1$$

- The distance between any two points a and b on the real line is

$$d(a, b) = |b - a| = |a - b|$$

Example 6

Find the distance between -8 and 2

Solution

$$D = |-8 - 2| = |-10| = 10$$

- A set المجموعة is a collection of objects called elements العناصر

$$A = \{1,2,3,4,5,6\}$$

لاحظ الأقواس المستخدمة { }

$$A = \{x | x \text{ is an integer and } 0 < x < 7\}$$

طريقة أخرى لكتابة

المجموعات تسمى

set-builder notation

- If A and B are sets, then their **union** اتحاد $A \cup B$ is the set that consists of all the elements that are in A and B

Example:

$$A = \{1,2,3,4,5\} \text{ and } B = \{4,5,6,7\}$$

$$A \cup B = \{1,2,3,4,5,6,7\}$$

- The **intersection** التقاطع of A and B is the set $A \cap B$ consisting of all elements that are in both A and B .

Example:

$$A = \{1,2,3,4,5\} \text{ and } B = \{4,5,6,7\}$$

$$A \cap B = \{4,5\}$$

- إذا كانت المجموعة ليس بها عناصر تسمى مجموعة خالية **empty set** و يرمز لها بالرمز $\emptyset$ (Phi)

Example:

$$A = \{1,2,3\} \text{ and } B = \{4,5,6,7\}$$

$$A \cap B = \emptyset$$

- **Intervals** الفترات are sets with an infinite number of elements and are defined by start and end values.

Open interval



$$(a, b) = \{x | a < x < b\}$$

Closed interval



$$[a, b] = \{x | a \leq x \leq b\}$$

Notation	Set description	Graph
(a, b)	$\{x a < x < b\}$	
$[a, b]$	$\{x a \leq x \leq b\}$	
$[a, b)$	$\{x a \leq x < b\}$	
$(a, b]$	$\{x a < x \leq b\}$	
(a, ∞)	$\{x a < x\}$	
$[a, \infty)$	$\{x a \leq x\}$	
$(-\infty, b)$	$\{x x < b\}$	
$(-\infty, b]$	$\{x x \leq b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

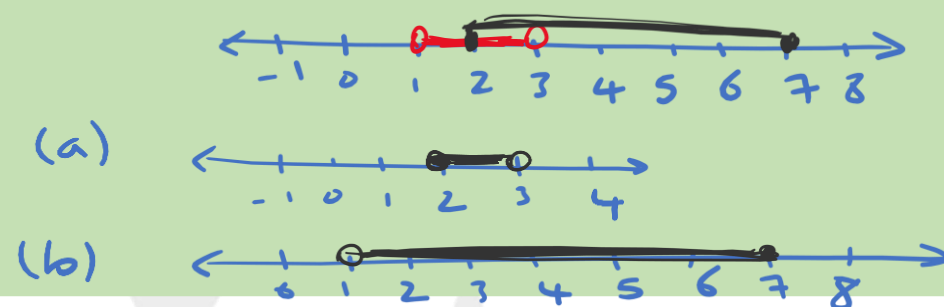
Example 7

Graph each set.

(a) $(1, 3) \cap [2, 7]$

(b) $(1, 3) \cup [2, 7]$

Solution



Problems

- Perform the indicated operations.

(a) $\frac{2}{\frac{2}{3}} - \frac{\frac{2}{3}}{2}$

$$\frac{2 \cdot 3}{\cancel{2}} - \frac{\cancel{2}}{2 \cdot 3}$$

$$= 3 - \frac{1}{3}$$

$$= \frac{9-1}{3} = \frac{8}{3}$$

مقام المقام يصار
مقام البسط ينزل

(b) $1 + \frac{5}{8} - \frac{1}{6}$

$$= \frac{8+5}{8} - \frac{1}{6} = \frac{13}{8} - \frac{1}{6}$$

$$= \frac{13 \cdot 6 - 1 \cdot 8}{8 \cdot 6} = \frac{78 - 8}{48}$$

$$= \frac{70}{48} = \frac{35}{24}$$

(c) $(3 + \frac{1}{4})(1 - \frac{4}{5})$

$$= 3 \cdot 1 + 3(-\frac{4}{5}) + \frac{1}{4} \cdot 1 + \frac{1}{4}(-\frac{4}{5})$$

$$= 3 - \frac{12}{5} + \frac{1}{4} - \frac{1}{5}$$

$$= \frac{3 \cdot 20 - 12 \cdot 4 + 1 \cdot 5 - 1 \cdot 4}{20} = \frac{60 - 48 + 5 - 4}{20}$$

$$= \frac{13}{20}$$

- Find the indicated set if

$$A = \{1, 2, 3, 4, 5, 6, 7\}, B = \{2, 4, 6, 8\}, \text{ and } C = \{7, 8, 9, 10\}$$

(a) $A \cup B \cup C$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

(b) $A \cap B \cap C = \emptyset$

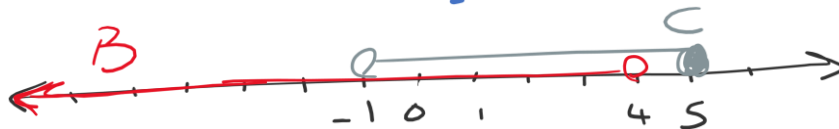
empty set

- Find the indicated set if

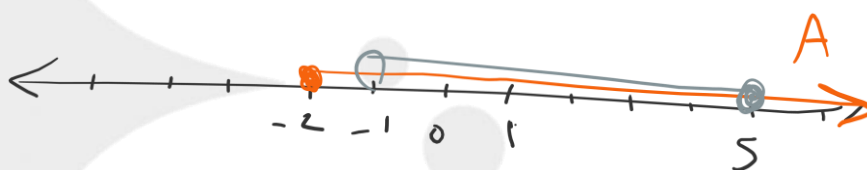
$$A = \{x | x \geq -2\}, B = \{x | x < 4\}, \text{ and } C = \{x | -1 < x \leq 5\}$$

(a) $B \cup C = \{x | x \leq 5\}$

يمكن رسم لتخيل الحل



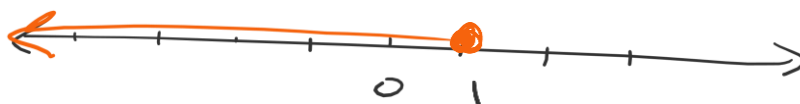
(b) $A \cap C = \{x | -1 < x \leq 5\}$



- Express the inequality in interval notation, and then graph the corresponding interval.

(a) $x \leq 1$

$$x = (-\infty, 1]$$



(b) $-2 < x \leq 1$

$$x = (-2, 1]$$



- Find the distance between the given numbers.

(a) -3 and 21

$$\begin{aligned} d &= |-3 - 21| \\ &= |-24| = 24 \end{aligned}$$

(b) $\frac{11}{8}$ and $-\frac{3}{10}$

$$\begin{aligned} d &= \left| \frac{11}{8} - \left(-\frac{3}{10}\right) \right| = \left| \frac{11}{8} + \frac{3}{10} \right| \\ &= \left| \frac{11 \cdot 10 + 3 \cdot 8}{80} \right| = \left| \frac{110 + 24}{80} \right| = \frac{134}{80} \end{aligned}$$

- Express each repeating decimal as a fraction

(a) $0.2\overline{8} = 0.28888 \dots$

$$\begin{aligned} 100x &= 28.88 \\ 10x &= 2.888 \\ \hline 90x &= 26 \\ \boxed{x &= \frac{26}{90}} \end{aligned}$$

(b) $5.\overline{23} = 5.232323 \dots$

$$\begin{aligned} 100x &= 523.23 \\ x &= 5.23 \\ \hline 99x &= 518 \\ \boxed{x &= \frac{518}{99}} \end{aligned}$$

(c) $2.1\overline{35} = 2.1353535 \dots$

$$\begin{aligned} 1000x &= 2135.35 \\ 10x &= 21.3535 \\ \hline 990x &= 2114 \\ \boxed{x &= \frac{2114}{990}} \end{aligned}$$