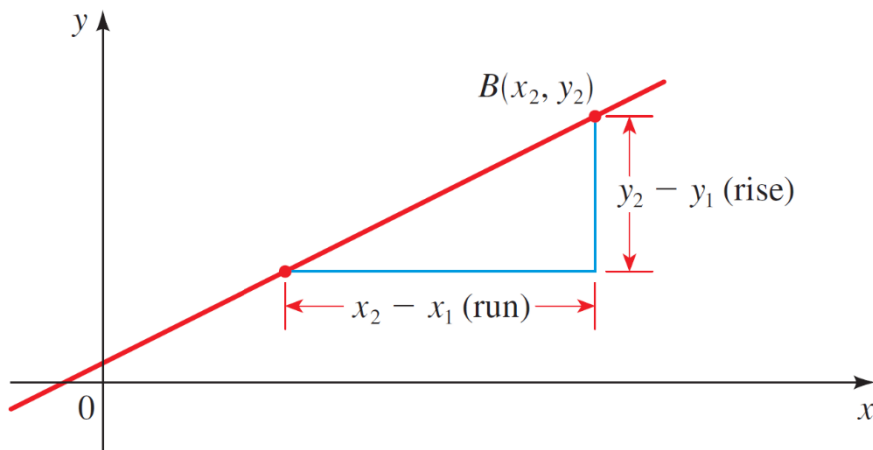


Section 1.10 – Lines

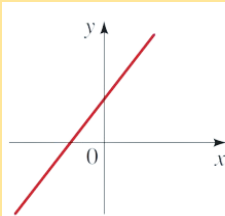
- The **slope** m of a line ميل الخط that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

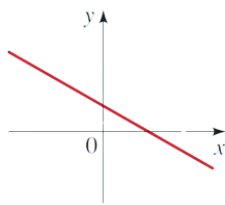


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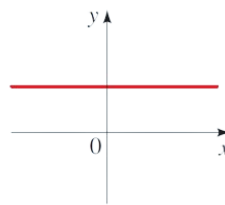
Positive slope



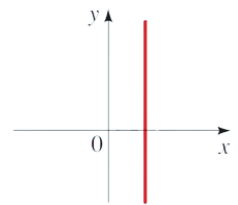
Negative slope



Zero slope



No slope



Example 1

Find the slope of the line that passes through the points $P(2,1)$ and $Q(8,5)$.

Solution

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 1}{8 - 2} = \frac{4}{6} = \frac{2}{3} \end{aligned}$$

Example 2

Find an equation of the line through the points $(-1, 2)$ and $(3, -4)$.

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{3 - (-1)} = \frac{-6}{4} = -\frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{2}(x - (-1))$$

$$y - 2 = -\frac{3}{2}x - \frac{3}{2}$$

$$y = -\frac{3}{2}x - \frac{3}{2} + 2 \Rightarrow y = -\frac{3}{2}x + \frac{1}{2}$$

Example 3

Find an equation of the line with slope 3 and y -intercept -2 .

Solution

$$y = mx + b$$

$$y = 3x - 2$$

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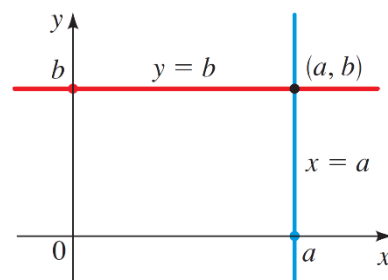
- Point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

- Slope-intercept form of the equation of a line:

$$y = mx + b$$

- An equation of the vertical line through (a, b) is $x = a$.
- An equation of the horizontal line through (a, b) is $y = b$.



Example 4

Find an equation of the line through $(1, 3)$ with slope 0

Solution

$$y = 3$$

- The graph of every linear equation

General equation of a line

$$Ax + By + C = 0 \quad (A, B \text{ not both zeros})$$

is a line. Its slope $m = -\frac{A}{B}$

Example 5

Find the slope of the line $2x - 3y - 12 = 0$.

Solution

$$m = -\frac{2}{-3}$$

$$m = \frac{2}{3}$$

- Two lines are **parallel** متوازية if and only if they have the same slope ($m_1 = m_2$)

Example 6

Find an equation of the line through the point (5,2) that is parallel to the line $4x + 6y + 5 = 0$

Solution

$$m_2 = m_1 = -\frac{4}{6} = -\frac{2}{3}$$

$$y - 2 = -\frac{2}{3}(x - 5)$$

$$y - 2 = -\frac{2}{3}x + \frac{10}{3}$$

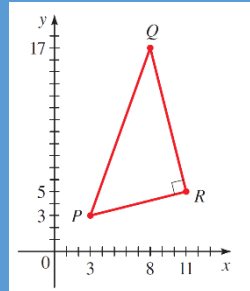
$$y = -\frac{2}{3}x + \frac{10}{3} + 2 \Rightarrow y = -\frac{2}{3}x + \frac{16}{3}$$

- Two lines are **perpendicular** متعامدة if and only if their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

Example 7

Show that the points $P(3,3)$, $Q(8,17)$, and $R(11,5)$ are the vertices of a right triangle.



Solution

$$m_{PR} = \frac{5 - 3}{11 - 3} = \frac{2}{8} = \frac{1}{4}$$

$$m_{QR} = \frac{5 - 17}{11 - 8} = \frac{-12}{3} = -4$$

since $m_{PR} = -\frac{1}{m_{QR}}$, then they are $\perp$

Problems

- Find an equation of the line with slope 2 and y-intercept -3 .

$$y = mx + b$$

$$y = 2x - 3$$

- Find an equation of the line through $(2, 3)$ with slope 5

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 5(x - 2)$$

$$y - 3 = 5x - 10$$

$$y = 5x - 10 + 3$$

$$y = 5x - 7$$

- Find an equation of the line through $(2, 1)$ and $(1, 6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6 - 1}{1 - 2} = \frac{5}{-1} = -5$$

$$y - 1 = -5(x - 2)$$

$$y - 1 = -5x + 10$$

$$y = -5x + 10 + 1$$

$$y = -5x + 11$$

- Find an equation of the line through $(2, -1)$ and slope undefined

$$x = 2$$

- Find an equation of the line through $(1, -6)$ and parallel to the line
 $x + 2y = 6$

$$2y = -x + 6$$

$$y = -\frac{1}{2}x + \frac{6}{2}$$

$$m_1 = m_2 = -\frac{1}{2}$$

$$y - (-6) = -\frac{1}{2}(x - 1)$$

$$y + 6 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{1}{2} - 6$$

$$y = -\frac{1}{2}x - \frac{11}{2}$$

- Find an equation of the line through $(-1, -2)$ and perpendicular to the line
 $2x + 5y + 8 = 0$

$$5y = -2x - 8$$

$$y = -\frac{2}{5}x - \frac{8}{5}$$

$$m_2 = -\frac{1}{m_1} = \frac{5}{2}$$

$$y - (-2) = \frac{5}{2}(x - (-1))$$

$$y + 2 = \frac{5}{2}x + \frac{5}{2}$$

$$y = \frac{5}{2}x + \frac{5}{2} - 2$$

$$y = \frac{5}{2}x + \frac{1}{2}$$