

Section 1.2 – Exponents and Radicals

- A product of identical numbers is written in exponential notation

حاصل ضرب رقم في نفسه يكتب على شكل أساس و أس

(عدد مرات التكرار)

Exponent الأس

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}$$

Base الأساس

Example 1

Find the values of the following exponents.

(a) $\left(\frac{1}{2}\right)^5$

(b) $(-3)^4$

(c) -3^4

Solution

$$(a) \left(\frac{1}{2}\right)^5 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32}$$

$$(b) (-3)^4 = (-3)(-3)(-3)(-3) = 81$$

$$(c) -3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$$

لاحظ

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1 \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

Example 2

Find the values of the following exponents.

(a) $\left(\frac{4}{7}\right)^0$

(b) $(-2)^{-3}$

Solution

(a) $\left(\frac{4}{7}\right)^0 = 1$

(b) $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$

- قوانين الأسس Laws of exponents

1. $a^m a^n = a^{m+n}$

$3^2 \cdot 3^5 = 3^{2+5} = 3^7$

2. $\frac{a^m}{a^n} = a^{m-n}$

$\frac{3^5}{3^2} = 3^{5-2} = 3^3$

3. $(a^m)^n = a^{mn}$

$(3^2)^5 = 3^{2 \cdot 5} = 3^{10}$

4. $(ab)^n = a^n b^n$

$(3 \cdot 4)^2 = 3^2 4^2$

5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

6. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$

7. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

$\frac{3^{-2}}{4^{-5}} = \frac{4^5}{3^2}$

Example 3

Simplify.

(a) $x^4 x^7$

(d) $(b^4)^5$

(b) $y^4 y^{-7}$

(e) $(3x)^3$

(c) $\frac{c^9}{c^5}$

(f) $\left(\frac{x}{2}\right)^5$

Solution

(a) $x^4 x^7 = x^{4+7} = x^{11}$

(b) $y^4 y^{-7} = y^{4-7} = y^{-3} = \frac{1}{y^3}$

(c) $\frac{c^9}{c^5} = c^{9-5} = c^4$

(d) $(b^4)^5 = b^{4 \cdot 5} = b^{20}$

(e) $(3x)^3 = 3^3 x^3 = 27x^3$

(f) $\left(\frac{x}{2}\right)^5 = \frac{x^5}{2^5} = \frac{x^5}{32}$

Example 4

Simplify $(2a^3b^2)(3ab^4)^3$

Solution

$$2a^3b^2(3^3a^3b^{4 \cdot 3})$$

$$= 2a^3b^2(27a^3b^{12})$$

$$= 2 \cdot 27 a^{3+3} b^{2+12}$$

$$= 54 a^6 b^{14}$$

- If n is a positive integer

$\sqrt[n]{a} = b$ means $b^n = a$

مهم جداً: إذا كان n عدد زوجي، لازم:
 $a \geq 0$ and $b \geq 0$

$\sqrt[4]{81} = 3$ because $3^4 = 81$ مثال

لاحظ

$\sqrt{a} = b$ means $b^2 = a$

جذر تربيعي "square root" وقيمة $n = 2$ و لا نحتاج أن نكتبها

- Properties of n th roots خواص الجذور

1. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ $\sqrt[3]{-8 \cdot 27} = \sqrt[3]{-8}\sqrt[3]{27} = (-2)(3) = -6$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$

3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ $\sqrt[3]{\sqrt[7]{729}} = \sqrt[21]{729} = 3$

4. $\sqrt[n]{a^n} = a$ if n is odd $\sqrt[3]{(-5)^3} = -5, \sqrt[5]{2^5} = 2$

5. $\sqrt[n]{a^n} = |a|$ if n is even $\sqrt[4]{(-5)^4} = |-5| = 5$

Example 5

Simplify $\sqrt[4]{81x^8y^4}$

Solution

$$\sqrt[4]{81} \sqrt[4]{x^8} \sqrt[4]{y^4}$$

$$= 3x^2y$$

لاحظ

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

Example 6

Simplify $\sqrt{32} + \sqrt{200}$

Solution

$$\begin{aligned} & \sqrt{16 \cdot 2} + \sqrt{100 \cdot 2} \\ &= \sqrt{16} \sqrt{2} + \sqrt{100} \sqrt{2} \\ &= 4\sqrt{2} + 10\sqrt{2} = 14\sqrt{2} \end{aligned}$$

- Rational exponents

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

where m and n are integers and $n > 0$

Note: If n is even زوجي then a must be ≥ 0

Example 7

Solve.

(a) $4^{1/2}$

(b) $8^{-2/3}$

Solution

$$(a) \quad 4^{1/2} = \sqrt[2]{4} = \pm 2$$

$$(b) \quad 8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$$

- عملية نقوم بها للتخلص من الجذور في المقام Rationalizing the denominator

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

تساوي 1

$$\frac{1}{\sqrt[n]{a^m}} = \frac{1}{\sqrt[n]{a^m}} \cdot \frac{\sqrt[n]{a^{n-m}}}{\sqrt[n]{a^{n-m}}} = \frac{\sqrt[n]{a^{n-m}}}{a}$$

Example 8

Put each fractional expression into standard form by rationalizing the denominator.

(a) $\frac{2}{\sqrt{3}}$

(b) $\frac{1}{\sqrt[3]{5}}$

(c) $\sqrt[7]{\frac{1}{a^2}}$

Solution

$$(a) \quad \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$(b) \quad \frac{1}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} = \frac{\sqrt[3]{25}}{5}$$

$$\begin{aligned} (c) \quad \sqrt[7]{\frac{1}{a^2}} &= \frac{\sqrt[7]{1}}{\sqrt[7]{a^2}} = \frac{1}{\sqrt[7]{a^2}} \\ &= \frac{1}{\sqrt[7]{a^2}} \cdot \frac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}} \\ &= \frac{\sqrt[7]{a^5}}{a} \end{aligned}$$

Problems

- Evaluate the expression -2^6

- Simplify, and eliminate any negative exponents.

(a) $x^{-5} \cdot x^3$

(b) $w^{-2}w^{-4}w^5$

(c) $\frac{x^{16}}{x^{10}}$

- Simplify, and eliminate any negative exponents.

(a) $(3x^3y^2)(2y^3)$

(b) $(5w^2z^{-2})^2(z^3)$

(c) $\frac{a^9a^{-2}}{a}$

- Simplify the expression.

(a) $\sqrt[6]{64a^6b^7}$

(b) $\sqrt[3]{a^2b} \sqrt[3]{64a^4b}$

(c) $\sqrt{32} + \sqrt{18}$

(d) $\sqrt{81x^2 + 81}$

- Evaluate each expression.

(a) $32^{2/5}$

(b) $\left(\frac{4}{9}\right)^{-1/2}$

(c) $\left(\frac{16}{81}\right)^{3/4}$

- Simplify the expression, and eliminate any negative exponent(s).

(a) $\frac{(8s^3t^3)^{2/3}}{(s^4t^{-8})^{1/4}}$

(b) $\left(\frac{x^{3/2}}{y^{-1/2}}\right)^4 \left(\frac{x^{-2}}{y^3}\right)$

- Simplify the expression, and eliminate any negative exponent(s).

(a) $\sqrt[6]{y^5} \sqrt[3]{y^2}$

(b) $\sqrt[6]{y\sqrt{y}}$

- Put each fractional expression into standard form by rationalizing the denominator.

(a) $\frac{1}{\sqrt{6}}$

(b) $\sqrt{\frac{3}{2}}$

(c) $\frac{9}{\sqrt[4]{2}}$