

Section 1.3 – Algebraic Expressions

- A **polynomial** الحدود كثيرة is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

↓ ↓ ↓ ↓
حدود الدالة
Terms

where $a_0, a_1, \dots, a_n$ are real numbers

لاحظ

Polynomial	Type نوعها	Terms حدودها	Degree درجتها (أكبر أس ل x)
$2x^2 - 3x + 4$	trinomial ثلاثية الحدود	$2x^2, -3x, 4$	2
$x^8 + 5x$	binomial ثنائية الحدود	$x^8, 5x$	8
$8 - x + x^2 - \frac{1}{2}x^3$	four terms	$-\frac{1}{2}x^3, x^2, -x, 8$	3
$5x + 1$	binomial	$5x, 1$	1
$9x^5$	monomial أحادية	$9x^5$	5
6	monomial	6	0

Example 1

- (a) Find the sum $(x^3 - 6x^2 + 2x + 4) + (x^3 + 5x^2 - 7x)$
 (b) Find the difference $(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x)$

Solution

$$\begin{aligned} \text{(a)} \quad & x^3 + x^3 - 6x^2 + 5x^2 + 2x - 7x + 4 \\ & = 2x^3 - x^2 - 5x + 4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \cancel{x^3} - \cancel{x^3} - 6x^2 - 5x^2 + 2x + 7x + 4 \\ & = -11x^2 + 9x + 4 \end{aligned}$$

Example 2

Find the product $(2x + 3)(x^2 - 5x + 4)$

Solution

$$\begin{aligned}
 & (2x + 3)(x^2 - 5x + 4) \\
 &= 2x^3 - \underline{10x^2} + \underline{8x} + \underline{3x^2} - \underline{15x} + 12 \\
 &= 2x^3 - 7x^2 - 7x + 12
 \end{aligned}$$

- بعض أشكال ضرب كثيرات الحدود تتكرر لدرجة أننا يجب أن نحفظها

$$1. (A + B)(A - B) = A^2 - B^2$$

$$2. (A + B)^2 = A^2 + 2AB + B^2$$

Square of sum

$$3. (A - B)^2 = A^2 - 2AB + B^2$$

Square of difference

$$4. (A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

Cube of sum

$$5. (A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$$

Cube of difference

Example 3

Find each product.

$$(a) (3x + 5)^2$$

$$(b) (x^2 - 2)^3$$

Solution

$$\begin{aligned}
 (a) (3x + 5)^2 &= (3x)^2 + 2(3x) \cdot 5 + 5^2 \\
 &= 9x^2 + 30x + 25
 \end{aligned}$$

$$\begin{aligned}
 (b) (x^2 - 2)^3 &= (x^2)^3 - 3(x^2)^2 \cdot 2 + 3(x^2) \cdot 2^2 - 2^3 \\
 &= x^6 - 6x^4 + 12x^2 - 8
 \end{aligned}$$

Example 4

Find each product.

(a) $(2x - \sqrt{y})(2x + \sqrt{y})$

(b) $(x + y - 1)(x + y + 1)$

Solution

$$\begin{aligned}
 \text{(a)} \quad (2x - \sqrt{y})(2x + \sqrt{y}) &= (2x)^2 - (\sqrt{y})^2 \\
 &= 4x^2 - y \\
 \text{(b)} \quad (x + y - 1)(x + y + 1) &= (x + y)^2 - 1^2 \\
 &= x^2 + 2xy + y^2 - 1
 \end{aligned}$$

- نستطيع تحليل (تفكيك) **factoring** كثيرات الحدود كما قمنا بتجميعها في الجزء السابق

■ تحليل (تفكيك) ➔
 $x^2 - 4 = (x - 2)(x + 2)$
➔ ضرب (تجميع) ■

Example 5

Factor each expression.

(a) $3x^2 - 6x$

(b) $8x^4y^2 + 6x^3y^3 - 2xy^4$

Solution

$$\begin{aligned}
 \text{(a)} \quad 3x^2 - 6x &= 3x(x - 2) \\
 \text{(b)} \quad 8x^4y^2 + 6x^3y^3 - 2xy^4 \\
 &= 2xy^2(4x^3 + 3x^2y - y^2)
 \end{aligned}$$

- تحليل ثلاثيات الحدود من الدرجة الثانية (x^2) Factoring Trinomials

$$c = r \times s$$

$$x^2 + bx + c = (x + r)(x + s)$$

$$b = r + s$$

بشرط أن معامل x يساوي 1

Example 5

Factor each expression.

(a) $x^2 - 7x + 12$

(b) $(5a + 1)^2 - 2(5a + 1) - 3$

(c) $6x^2 + 7x - 5$

Solution

$$(a) \quad x^2 - 7x + 12 = (x - 4)(x - 3)$$

$$(b) \quad (5a + 1)^2 - 2(5a + 1) - 3$$

$$= ((5a + 1) - 3)((5a + 1) + 1)$$

$$= (5a - 2)(5a + 2)$$

$$(c) \quad 6x^2 + 7x - 5 = (3x + 5)(2x - 1)$$

- بعض كثيرات الحدود لها قواعد ثابتة في التحليل

$$1. A^2 - B^2 = (A + B)(A - B)$$

فرق بين مربعين Difference of squares

$$2. A^2 + 2AB + B^2 = (A + B)^2$$

مربع كامل
Perfect square }
Perfect square

$$3. A^2 - 2AB + B^2 = (A - B)^2$$

$$4. A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

فرق بين مكعبين Difference of cubes

$$5. A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

مجموع مكعبين Sum of cubes

Example 6

Factor each expression.

(a) $4x^2 - 25$

(b) $(x + y)^2 - z^2$

(c) $x^2 + 6x + 9$

(d) $4x^2 - 4xy + y^2$

(e) $27x^3 - 1$

Solution

$$(a) \quad 4x^2 - 25 = (2x - 5)(2x + 5)$$

$$(b) \quad (x + y)^2 - z^2 = ((x + y) - z)((x + y) + z)$$

$$(c) \quad x^2 + 6x + 9 = (x + 3)^2$$

$$(d) \quad 4x^2 - 4xy + y^2 = (2x - y)^2$$

$$(e) \quad 27x^3 - 1 = (3x - 1)(9x^2 + 3x + 1)$$

Example 7

Factor each expression.

(a) $2x^4 - 8x^2$

(b) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

(c) $x^3 + x^2 + 4x + 4$

Solution

$$\begin{aligned} \text{(a)} \quad 2x^4 - 8x^2 &= 2x^2(x^2 - 4) \\ &= 2x^2(x - 2)(x + 2) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3x^{3/2} - 9x^{1/2} + 6x^{-1/2} \\ &= 3x^{-1/2}(x^2 - 3x + 2) \\ &= 3x^{-1/2}(x - 2)(x - 1) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad x^3 + x^2 + 4x + 4 \\ &= x^2(x + 1) + 4(x + 1) \\ &= (x + 1)(x^2 + 4) \end{aligned}$$

Problems

- Find the sum, difference, or product.

(a) $(-2x^2 - 3x + 1) + (3x^2 + 5x - 4)$

$$= -2x^2 + 3x^2 - 3x + 5x + 1 - 4$$

$$= x^2 + 2x - 3$$

(b) $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2)$

$$= 5x^3 + 4x^2 - x^2 - 3x - 7x - 2$$

$$= 5x^3 + 3x^2 - 10x - 2$$

(c) $(3t - 2)(7t - 4)$

$$= 21t^2 - 12t - 14t + 8$$

$$= 21t^2 - 26t + 8$$

- Multiply the algebraic expressions.

(a) $(5x + 1)^2$

$$= (5x)^2 + 2 \cdot 5x \cdot 1 + 1^2$$

$$= 25x^2 + 10x + 1$$

square of sum

(b) $(y + 2)^3$

$$= y^3 + 3y^2 \cdot 2 + 3y \cdot 2^2 + 2^3$$

$$= y^3 + 6y^2 + 12y + 8$$

cube of sum

- Perform the indicated operations and simplify.

(a) $(x + 2)(x^2 + 2x + 3)$

$$= x^3 + \underline{2x^2} + \underline{3x} + \underline{2x^2} + \underline{4x} + 6$$

$$= x^3 + 4x^2 + 7x + 6$$

(b) $(\sqrt{a} - b)(\sqrt{a} + b)$

$$= a - b^2$$

(c) $(2x + y - 3)(2x + y + 3)$

$$= ((2x + y) - 3)((2x + y) + 3)$$

$$= (2x + y)^2 - 9$$

$$= (2x)^2 + 2 \cdot 2x \cdot y + y^2 - 9$$

$$= 4x^2 + 4xy + y^2 - 9$$

- Factor completely.

(a) $-2x^3 + x$

$$= -x (2x^2 - 1)$$

(b) $y(y - 6) + 9(y - 6)$

$$= (y - 6)(y + 9)$$

(c) $x^2 + 8x + 7$

$$= (x + 7)(x + 1)$$

(d) $8x^2 - 14x - 15$

$$(4x + 3)(2x - 5)$$

$$\begin{array}{r} 8x^2 \\ -20x \\ +6x \\ -15 \\ \hline 8x^2 - 14x - 15 \end{array}$$

(e) $(3x+2)^2 - 8(3x+2) + 12$

$$3x+2 = w$$

$$w^2 - 8w + 12$$

$$= (w-6)(w-2)$$

$$\therefore (3x+2)^2 - 8(3x+2) + 12$$

$$= ((3x+2)-6)((3x+2)-2)$$

$$= (3x-4)(3x)$$

(f) $9a^2 - 16$

$$= (3a)^2 - 4^2$$

$$= (3a-4)(3a+4)$$

(g) $27x^3 + y^3$

$$= (3x)^3 + y^3$$

$$= (3x+y)((3x)^2 - 3xy + y^2)$$

$$= (3x+y)(9x^2 - 3xy + y^2)$$

$$(h) x^3 + 4x^2 + x + 4$$

$$= x^2(x + 4) + x + 4$$

$$= (x + 4)(x^2 + 1)$$

$$(i) (x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2}$$

$$= (x^2 + 1)^{-1/2} ((x^2 + 1) + 2)$$

$$= (x^2 + 1)^{-1/2} (x^2 + 3)$$

$$(j) x^3 + 2x^2 + x$$

$$= x(x^2 + 2x + 1)$$

$$= x(x+1)^2$$

$$(k) x^4y^3 - x^2y^5$$

$$= x^2y^3(x^2 - y^2)$$

$$= x^2y^3(x-y)(x+y)$$