

Section 1.4 – Rational Expressions

- A quotient تقسيم of two algebraic expressions is called a fractional expression

$$\frac{x}{\sqrt{x^2 + 1}}$$

- إذا كان البسط والمقام بدون جذور يسمى **rational expression**

$$\frac{2x}{x-1}$$

$$\frac{y-2}{y^2+4}$$

$$\frac{x^3-x}{x^2-5x+6}$$

- The domain المجال of fractional expressions are all values of x such that the denominator

المقام $\neq 0$

كل الأعداد ماعدا 1

Example: $\frac{1}{x-1}$ D: $\{x | x \neq 1\}$

Example 1

Find the domain of the following expressions.

(a) $\frac{\sqrt{x}}{x-5}$

(b) $\frac{x-5}{\sqrt{x}}$

(c) $\frac{x}{x^2-5x+6}$

Solution

(a)

(b) $\sqrt{x} = 0 \Rightarrow x = 0$

D: $\{x | x > 0\}$

(c) $x^2 - 5x + 6 = 0 \Rightarrow (x-3)(x-2) = 0$
 $x = 3 \quad x = 2$

D: $\{x | x \neq 2 \text{ and } x \neq 3\}$

Example 2

Simplify the rational expressions $\frac{x^2-1}{x^2+x-2}$

Solution

$$\frac{\cancel{(x-1)}(x+1)}{(x+2)\cancel{(x-1)}} = \frac{x+1}{x+2}$$

- Multiplying and dividing rational expressions

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

Example 3

Perform the multiplication and simplify $\frac{x^2+2x-3}{x^2+8x+16} \cdot \frac{3x+12}{x-1}$

Solution

$$\frac{(x+3)\cancel{(x-1)}}{(x+4)\cancel{(x+4)}} \cdot \frac{3\cancel{(x+4)}}{\cancel{x-1}}$$

$$\frac{3(x+3)}{x+4}$$

Example 4

Perform the division and simplify $\frac{x-4}{x^2-4} \div \frac{x^2-3x-4}{x^2+5x+6}$

Solution

$$\begin{aligned} & \frac{x-4}{x^2-4} \cdot \frac{x^2+5x+6}{x^2-3x-4} \\ &= \frac{\cancel{x-4}}{\cancel{(x+2)}(x-2)} \cdot \frac{(x+3)\cancel{(x+2)}}{\cancel{(x-4)}(x+1)} \\ &= \frac{x+3}{(x-2)(x+1)} \end{aligned}$$

- Adding and subtracting rational expressions

$$\frac{A}{B} \pm \frac{C}{D} = \frac{AD \pm CB}{BD}$$

Example 5

Perform the indicated operation and simplify $\frac{3}{x-1} + \frac{3}{x+2}$

Solution

$$\begin{aligned} \frac{3(x+2)+3(x-1)}{(x-1)(x+2)} &= \frac{3x+6+3x-3}{(x-1)(x+2)} \\ &= \frac{6x+3}{(x-1)(x+2)} = \frac{3(2x+1)}{(x-1)(x+2)} \end{aligned}$$

لاحظ

$$\frac{A}{B+C} \neq \frac{A}{B} + \frac{A}{C}$$

لا يمكن تقسيم الجمع أو الطرح في المقام

Example 6

Simplify

(a) $\frac{\frac{x}{y} + 1}{1 - \frac{y}{x}}$

(b) $\frac{\frac{1}{a+h} - \frac{1}{a}}{h}$

Solution

$$(a) \left(\frac{x}{y} + 1 \right) \div \left(1 - \frac{y}{x} \right)$$

$$= \left(\frac{x}{y} + \frac{y}{y} \right) \div \left(\frac{x}{x} - \frac{y}{x} \right)$$

$$= \frac{x+y}{y} \div \frac{x-y}{x}$$

$$= \frac{x+y}{y} \cdot \frac{x}{x-y} = \frac{x(x+y)}{y(x-y)}$$

$$(b) \left(\frac{1}{a+h} - \frac{1}{a} \right) \div h$$

$$= \frac{a - (a+h)}{(a+h)a} \div h = \frac{a - a - h}{a(a+h)} \cdot \frac{1}{h}$$

$$= \frac{-\cancel{h}}{a(a+h)} \cdot \frac{1}{\cancel{h}} = \frac{-1}{a(a+h)}$$

- Rationalizing the denominator or the numerator

إذا كان البسط أو المقام على شكل $A + B\sqrt{C}$

نستطيع التخلص من الجذر بالضرب في المرافق conjugate

$$(A + B\sqrt{C})(A - B\sqrt{C}) = A^2 - B^2C$$

Example 7

Rationalize the denominator or the numerator.

- (a) $\frac{1}{1+\sqrt{2}}$
 (b) $\frac{\sqrt{4+h}-2}{h}$

Solution

$$\begin{aligned} \text{(a)} \quad & \frac{1}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} \\ &= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = -1+\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} \\ &= \frac{\cancel{4+h}-4}{h(\sqrt{4+h}+2)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)} \\ &= \frac{1}{\sqrt{4+h}+2} \end{aligned}$$

Problems

- Find the domain of the expression.

(a) $\frac{x^2+1}{x^2-x-2}$

(b) $\frac{\sqrt{x}}{x+1}$

- Simplify the rational expression.

(a) $\frac{x^2+5x+6}{x^2+8x+15}$

(b) $\frac{1-x^2}{x^3-1}$

- Perform the multiplication or division and simplify.

(a) $\frac{x^2+2x-15}{x^2-25} \cdot \frac{x-5}{x+3}$

(b) $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$

- Perform the addition or subtraction and simplify.

(a) $\frac{3}{x+1} - \frac{1}{x+2}$

(b) $\frac{5}{2x-3} - \frac{3}{(2x-3)^2}$

- Simplify the compound fractional expression

(a) $\frac{1 + \frac{1}{x}}{\frac{1}{x} - 2}$

(b) $\frac{\frac{1}{1+x+h} - \frac{1}{1+x}}{h}$

- Rationalize the denominator $\frac{1}{5-\sqrt{3}}$

- Rationalize the numerator $\frac{1-\sqrt{5}}{3}$