Section 1.4 – Rational Expressions

- A quotient تقسيم of two algebraic expressions is called a fractional expression

$$\frac{x}{\sqrt{x^2+1}}$$

-إذا كان البسط والمقام بدون جذور يسمى rational expression

$$\frac{2x}{x-1} \qquad \frac{y-2}{y^2+4} \qquad \frac{x^3-x}{x^2-5x+6}$$

- The domain المجال of fractional expressions are all values of x such that the denominator

المقام $\neq 0$

Example: $\frac{1}{x-1}$ D: $\{x|x \neq 1\}$ كل الأعداد ماعدا 1

Example 1

Find the domain of the following expressions.

(a) $\frac{\sqrt{x}}{x-5}$

(b) $\frac{x-5}{\sqrt{x}}$

(c) $\frac{x}{x^2-5x+6}$

Solution

(a)

(b) $\sqrt{x} = 0 \Rightarrow x = 0$

D: $\{x | x > 0\}$

(c) $x^2 - 5x + 6 = 0 \Rightarrow (x-3)(x-2) = 0$

$x = 3 \quad x = 2$

D: $\{x | x \neq 2$ and $x \neq 3\}$

Example 2

Simplify the rational expressions $\frac{x^2-1}{x^2+x-2}$

Solution

$$\frac{\cancel{(x-1)}(x+1)}{(x+2)\cancel{(x-1)}} = \frac{x+1}{x+2}$$

- Multiplying and dividing rational expressions

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

Example 3

Perform the multiplication and simplify $\frac{x^2+2x-3}{x^2+8x+16} \cdot \frac{3x+12}{x-1}$

Solution

$$\frac{(x+3)\cancel{(x-1)}}{(x+4)\cancel{(x+4)}} \cdot \frac{3\cancel{(x+4)}}{\cancel{x-1}}$$

$$\frac{3(x+3)}{x+4}$$

Example 4

Perform the division and simplify $\frac{x-4}{x^2-4} \div \frac{x^2-3x-4}{x^2+5x+6}$

Solution

$$\frac{x-4}{x^2-4} \cdot \frac{x^2+5x+6}{x^2-3x-4}$$

$$= \frac{\cancel{x-4}}{\cancel{(x+2)}(x-2)} \cdot \frac{(x+3)\cancel{(x+2)}}{\cancel{(x-4)}(x+1)}$$

$$= \frac{x+3}{(x-2)(x+1)}$$

- Adding and subtracting rational expressions

$$\frac{A}{B} \pm \frac{C}{D} = \frac{AD \pm CB}{BD}$$

Example 5

Perform the indicated operation and simplify $\frac{3}{x-1} + \frac{3}{x+2}$

Solution

$$\frac{3(x+2)+3(x-1)}{(x-1)(x+2)} = \frac{3x+6+3x-3}{(x-1)(x+2)}$$

$$= \frac{6x+3}{(x-1)(x+2)} = \frac{3(2x+1)}{(x-1)(x+2)}$$

لاحظ

لا يمكن تقسيم الجمع أو الطرح في المقام

$$\frac{A}{B+C} \neq \frac{A}{B} + \frac{A}{C}$$

Example 6

Simplify

(a) $\dfrac{\frac{x}{y}+1}{1-\frac{y}{x}}$

(b) $\dfrac{\frac{1}{a+h}-\frac{1}{a}}{h}$

Solution

(a) $\left(\frac{x}{y}+1\right)\div\left(1-\frac{y}{x}\right)$

$=\left(\frac{x}{y}+\frac{y}{y}\right)\div\left(\frac{x}{x}-\frac{y}{x}\right)$

$=\dfrac{x+y}{y}\div\dfrac{x-y}{x}$

$=\dfrac{x+y}{y}\cdot\dfrac{x}{x-y}=\dfrac{x(x+y)}{y(x-y)}$

(b) $\left(\frac{1}{a+h}-\frac{1}{a}\right)\div h$

$=\dfrac{a-(a+h)}{(a+h)a}\div h=\dfrac{a-a-h}{a(a+h)}\cdot\dfrac{1}{h}$

$=\dfrac{-\cancel{h}}{a(a+h)}\cdot\dfrac{1}{\cancel{h}}=\dfrac{-1}{a(a+h)}$

- Rationalizing the denominator or the numerator

إذا كان البسط أو المقام على شكل $A + B\sqrt{C}$

نستطيع التخلص من الجذر بالضرب في المرافق conjugate

$$(A + B\sqrt{C})(A - B\sqrt{C}) = A^2 - B^2 C$$

Example 7

Rationalize the denominator or the numerator.

(a) $\frac{1}{1+\sqrt{2}}$

(b) $\frac{\sqrt{4+h}-2}{h}$

Solution

(a) $\frac{1}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = -1+\sqrt{2}$

(b) $\frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \frac{(\cancel{4}+h)-\cancel{4}}{h(\sqrt{4+h}+2)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$

$= \frac{1}{\sqrt{4+h}+2}$

Problems

- Find the domain of the expression.

(a) $\frac{x^2+1}{x^2-x-2}$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \qquad x = -1$

$D: \mathbb{R} // \{-1, 2\}$

$= \{x \mid x \neq -1, x \neq 2\}$

$= (-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

(b) $\frac{\sqrt{x}}{x+1}$

$x \neq -1$ and $x \geq 0$

صفر المقام   الجذر

$D: [0, \infty) \Leftarrow$ تقاطع الشرطين

$= \{x \mid x \geq 0\}$

- Simplify the rational expression.

(a) $\frac{x^2+5x+6}{x^2+8x+15}$

$$= \frac{\cancel{(x+3)}(x+2)}{\cancel{(x+3)}(x+5)}$$

$$= \frac{x+2}{x+5}$$

(b) $\frac{1-x^2}{x^3-1}$

$$= \frac{(1-x)(1+x)}{(x-1)(x^2+x+1)}$$

$$= \frac{-\cancel{(x-1)}(1+x)}{\cancel{(x-1)}(x^2+x+1)}$$

$$= \frac{-(1+x)}{x^2+x+1}$$

- Perform the multiplication or division and simplify.

(a) $\frac{x^2+2x-15}{x^2-25} \cdot \frac{x-5}{x+3}$

$$= \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}\cancel{(x-5)}} \cdot \frac{\cancel{x-5}}{x+3}$$

$$= \frac{x-3}{x+3}$$

(b) $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$

$$= \frac{2x+1}{2x^2+x-15} \cdot \frac{x+3}{6x^2-x-2}$$

$$= \frac{\cancel{2x+1}}{(2x-5)\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{\cancel{(2x+1)}(3x-2)}$$

$$= \frac{1}{(2x-5)(3x-2)}$$

- Perform the addition or subtraction and simplify.

(a) $\frac{3}{x+1} - \frac{1}{x+2}$

$$= \frac{3(x+2) - 1(x+1)}{(x+1)(x+2)}$$

$$= \frac{3x + 6 - x - 1}{(x+1)(x+2)}$$

$$= \frac{2x + 5}{(x+1)(x+2)}$$

(b) $\frac{5}{2x-3} - \frac{3}{(2x-3)^2}$

$$= \frac{5(2x-3) - 3}{(2x-3)^2}$$

$$= \frac{10x - 15 - 3}{(2x-3)^2}$$

$$= \frac{10x - 18}{(2x-3)^2} = \frac{2(5x-9)}{(2x-3)^2}$$

- Simplify the compound fractional expression

(a) $\dfrac{1+\frac{1}{x}}{\frac{1}{x}-2}$

$$= \left(1+\frac{1}{x}\right) \div \left(\frac{1}{x}-2\right)$$

$$= \left(\frac{x+1}{x}\right) \div \left(\frac{1-2x}{x}\right)$$

$$= \frac{x+1}{\cancel{x}} \cdot \frac{\cancel{x}}{1-2x}$$

$$= \frac{x+1}{1-2x}$$

(b) $\dfrac{\frac{1}{1+x+h}-\frac{1}{1+x}}{h}$

$$= \left(\frac{1}{1+x+h} - \frac{1}{x+h}\right) \div h$$

$$= \frac{(x+h)-(1+x+h)}{(1+x+h)(x+h)} \cdot \frac{1}{h}$$

$$= \frac{\cancel{x}+\cancel{h}-1-\cancel{x}-\cancel{h}}{h(1+x+h)(x+h)}$$

$$= \frac{-1}{h(1+x+h)(x+h)}$$

- Rationalize the denominator $\frac{1}{5-\sqrt{3}}$

$$\frac{1}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

$$= \frac{1(5+\sqrt{3})}{25-3}$$

$$= \frac{5+\sqrt{3}}{22}$$

- Rationalize the numerator $\frac{1-\sqrt{5}}{3}$

$$\frac{1-\sqrt{5}}{3} \cdot \frac{1+\sqrt{5}}{1+\sqrt{5}}$$

$$= \frac{1-5}{3(1+\sqrt{5})}$$

$$= \frac{-4}{3+3\sqrt{5}}$$