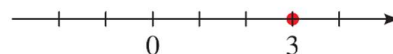


Section 1.8 – Inequalities

- Equations المعادلات have a finite number of solutions عدد محدود من الحلول, while **inequalities** لها عدد لا نهائي من الحلول have infinite number of solutions المتباينات.

Equation: $4x + 7 = 19$ **solution** $x = 3$



Inequality: $4x + 7 \leq 19$ $x \leq 3$



Rules for inequalities

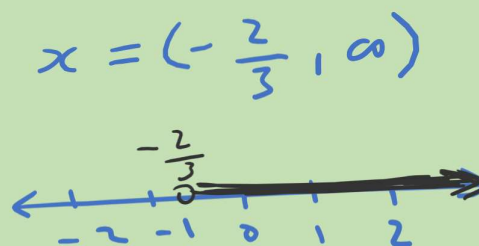
1. $A \leq B \Leftrightarrow A + C \leq B + C$
2. $A \leq B \Leftrightarrow A - C \leq B - C$
3. If $C > 0$, then $A \leq B \Leftrightarrow CA \leq CB$
4. If $C < 0$, then $A \leq B \Leftrightarrow CA \geq CB$
5. If $A > 0$ and $B > 0$, then $A \leq B \Leftrightarrow \frac{1}{A} \geq \frac{1}{B}$
6. If $A \leq B$ and $C \leq D$, then $A + C \leq B + D$
7. If $A \leq B$ and $B \leq C$, then $A \leq C$

Example 1

Solve the inequality $3x < 9x + 4$, and sketch the solution set.

Solution

$$\begin{aligned}
 3x - 9x &< 4 \\
 -6x &< 4 \\
 x &> \frac{4}{-6} \\
 x &> -\frac{2}{3}
 \end{aligned}$$



Example 2

Solve the inequality $4 \leq 3x - 2 < 13$.

Solution

$$4 + 2 \leq 3x < 13 + 2$$

$$\frac{6}{3} \leq x < \frac{15}{3}$$

$$2 \leq x < 5$$

$$x = [2, 5)$$



Example 3

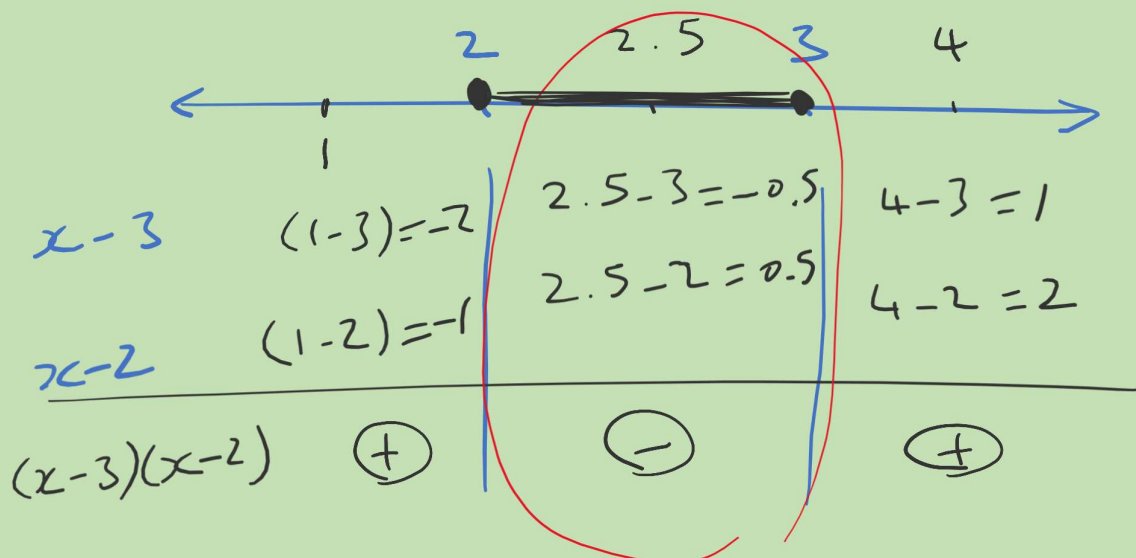
Solve the inequality $x^2 \leq 5x - 6$.

Solution

$$x^2 - 5x + 6 \leq 0$$

$$(x - 3)(x - 2) \leq 0$$

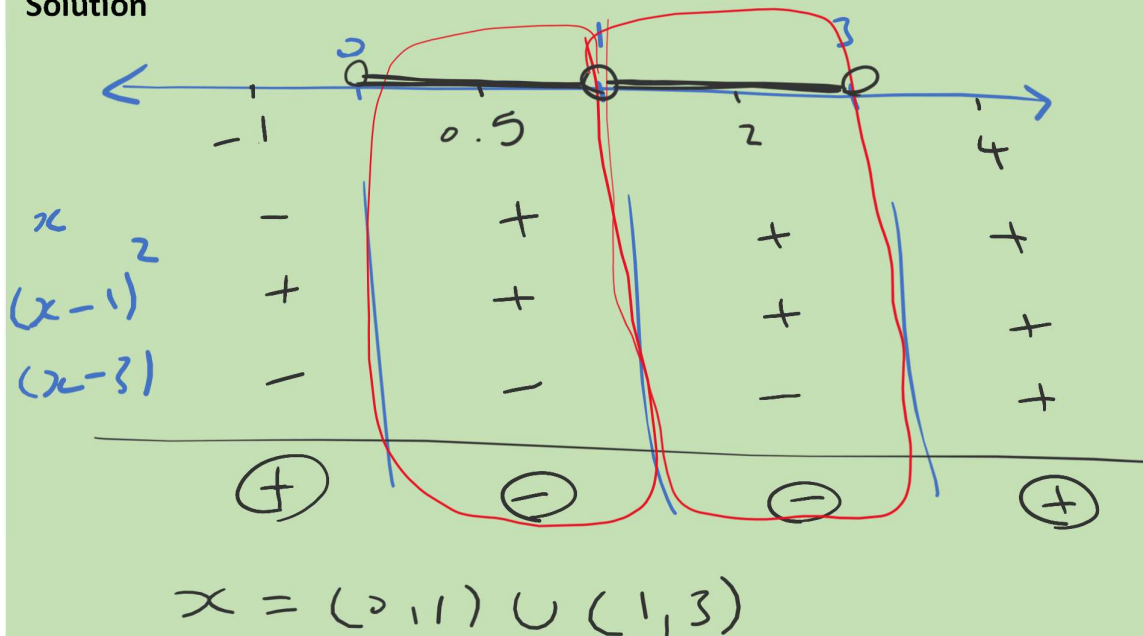
$$x = [2, 3]$$



Example 4

Solve the inequality $x(x-1)^2(x-3) < 0$.

Solution



Example 5

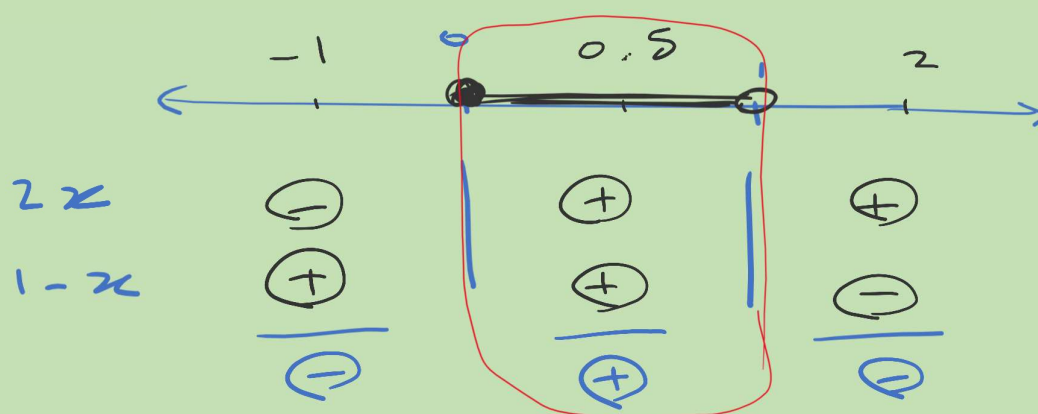
Solve the inequality $\frac{1+x}{1-x} \geq 1$.

Solution

$$\begin{aligned} \frac{1+x}{1-x} - 1 &\geq 0 \\ \frac{1+x}{1-x} - \frac{1-x}{1-x} &\geq 0 \\ \frac{1+x-(1-x)}{1-x} &\geq 0 \\ \frac{2x}{1-x} &\geq 0 \end{aligned}$$

لاحظ

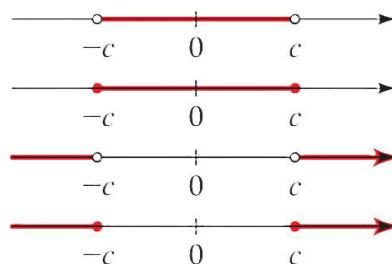
لا يمكن ضرب طرفي المعادلة في $(1-x)$ للتخلص من المقام لأننا لا نعرف إذا كانت قيمتها سالبة وبالتالي إذا يجب عكس اتجاه المتباينة inequality

Solution

$$x = [0, 0.5)$$

- Properties of absolute value inequalities

Inequality	Equivalent form
1. $ x < c$	$-c < x < c$
2. $ x \leq c$	$-c \leq x \leq c$
3. $ x > c$	$x < -c$ or $x > c$
4. $ x \geq c$	$x \leq -c$ or $x \geq c$



Example 6

Solve the inequality $|x - 5| < 2$.

Solution

$$-2 < x - 5 < 2$$

$$3 < x < 7$$

$$x = (3, 7)$$

Example 7

Solve the inequality $|3x + 2| \geq 4$.

Solution

$$3x + 2 \leq -4$$

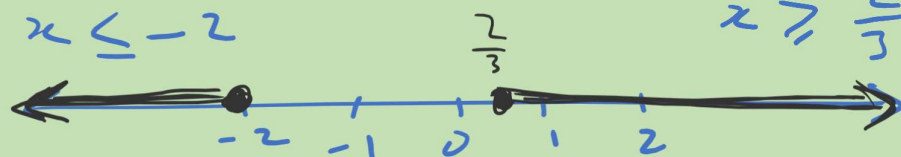
$$3x \leq -6$$

$$x \leq -2$$

$$3x + 2 \geq 4$$

$$3x \geq 2$$

$$x \geq \frac{2}{3}$$



$$x = (-\infty, -2] \cup [\frac{2}{3}, \infty)$$

Problems

- Solve the linear inequality. Express the solution using interval notation and graph the solution set.

(a) $4x - 7 < 8 + 9x$

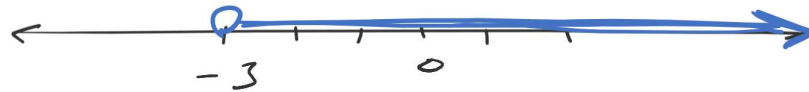
$$4x - 9x < 8 + 7$$

$$-5x < 15$$

$$x > \frac{15}{-5}$$

$$x > -3$$

$$x = (-3, \infty)$$



(b) $-2 < 8 - 2x \leq -1$

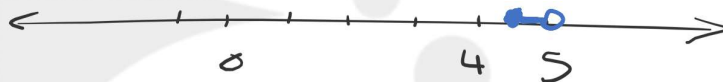
$$-2 - 8 < -2x \leq -1 - 8$$

$$-10 < -2x \leq -9$$

$$-\frac{10}{-2} < x \leq \frac{-9}{-2}$$

$$5 < x \leq 4\frac{1}{2}$$

$$x = [4\frac{1}{2}, 5)$$



- Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

(a) $2x^2 + x \geq 1$

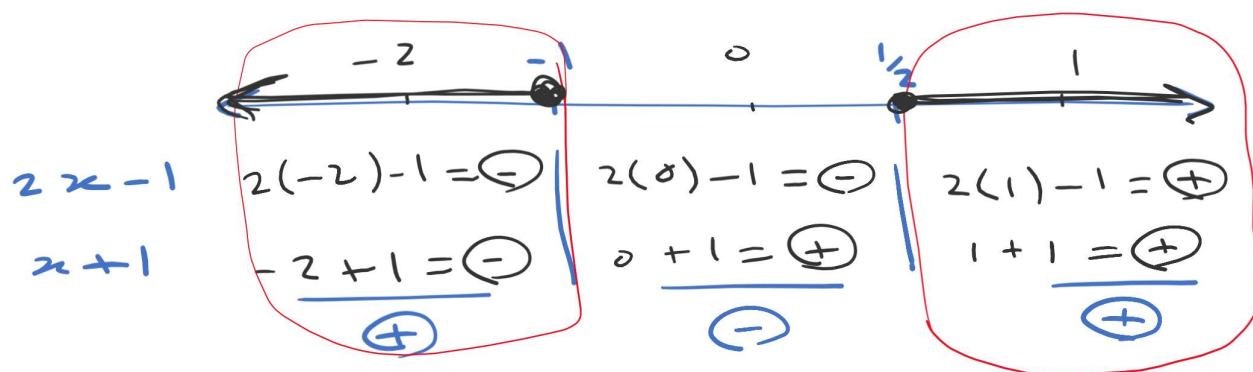
$$2x^2 + x - 1 \geq 0$$

$$(2x-1)(x+1) \geq 0$$

zeros

$$x+1=0 \Rightarrow x=-1$$

$$2x-1=0 \Rightarrow x=\frac{1}{2}$$



$$x = (-\infty, -1] \cup [\frac{1}{2}, \infty)$$

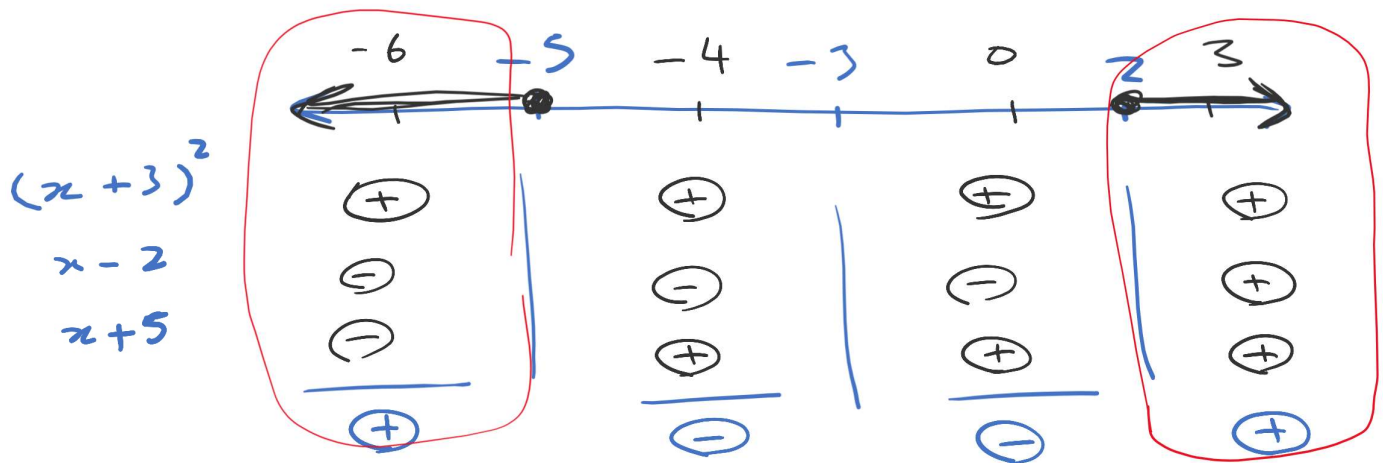
(b) $(x + 3)^2(x - 2)(x + 5) \geq 0$

zeros

$$x + 3 = 0 \Rightarrow x = -3$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x + 5 = 0 \Rightarrow x = -5$$

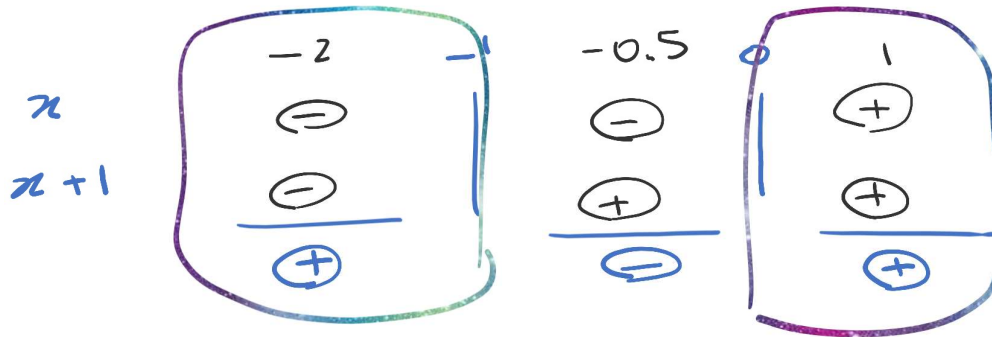


$$x = (-\infty, -5] \cup [2, \infty)$$

- Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

(a) $\frac{x}{x+1} > 3$

zeros
 $x = 0$ and $x = -1$



$$x = (-\infty, -1) \cup (0, \infty)$$



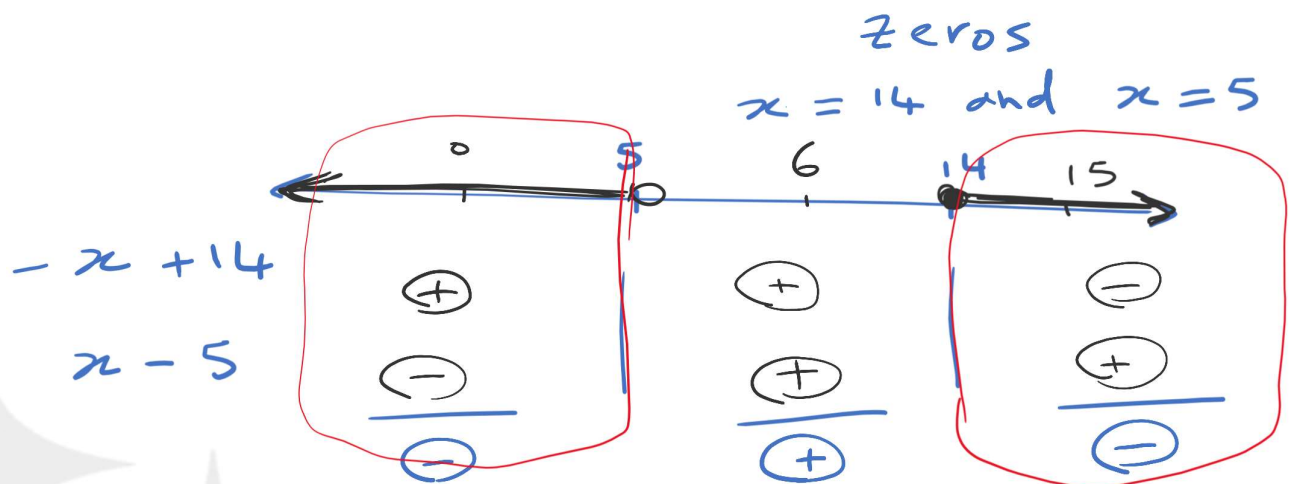
(b) $\frac{2x+1}{x-5} \leq 3$

$$\frac{2x-1}{x-5} - 3 \leq 0$$

$$\frac{2x-1-3(x-5)}{x-5} \leq 0$$

$$\frac{2x-1-3x+15}{x-5} \leq 0$$

$$\frac{-x+14}{x-5} \leq 0$$



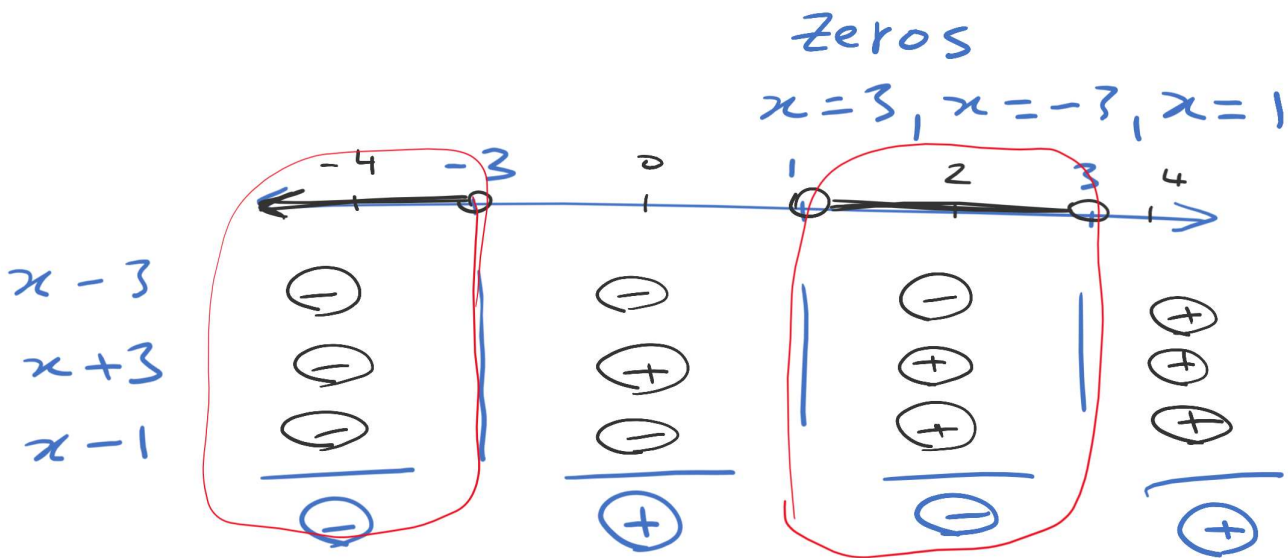
$$x = (-\infty, 5) \cup [14, \infty)$$

↓
استثنى 5 لأننا
نفس القوم

- Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

$$\frac{x^2 - 9}{x - 1} < 0$$

$$\frac{(x-3)(x+3)}{x-1} < 0$$



$$x = (-\infty, -3) \cup (1, 3)$$

- Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

(a) $|3x + 2| < 4$

$$-4 < 3x + 2 < 4$$

$$-4 - 2 < 3x < 4 - 2$$

$$-6 < 3x < 2$$

$$-\frac{6}{3} < x < \frac{2}{3}$$

$$-2 < x < \frac{2}{3}$$

$$x = (-2, \frac{2}{3})$$



(b) $|3x - 2| \geq 5$

$$3x - 2 \leq -5 \quad \text{or} \quad 3x - 2 \geq 5$$

$$3x \leq -5 + 2$$

$$3x \leq -3$$

$$x \leq -\frac{3}{3}$$

$$x \leq -1$$

$$3x - 2 \geq 5$$

$$3x \geq 5 + 2$$

$$3x \geq 7$$

$$x \geq \frac{7}{3}$$

$$x = (-\infty, -1] \cup [\frac{7}{3}, \infty)$$

