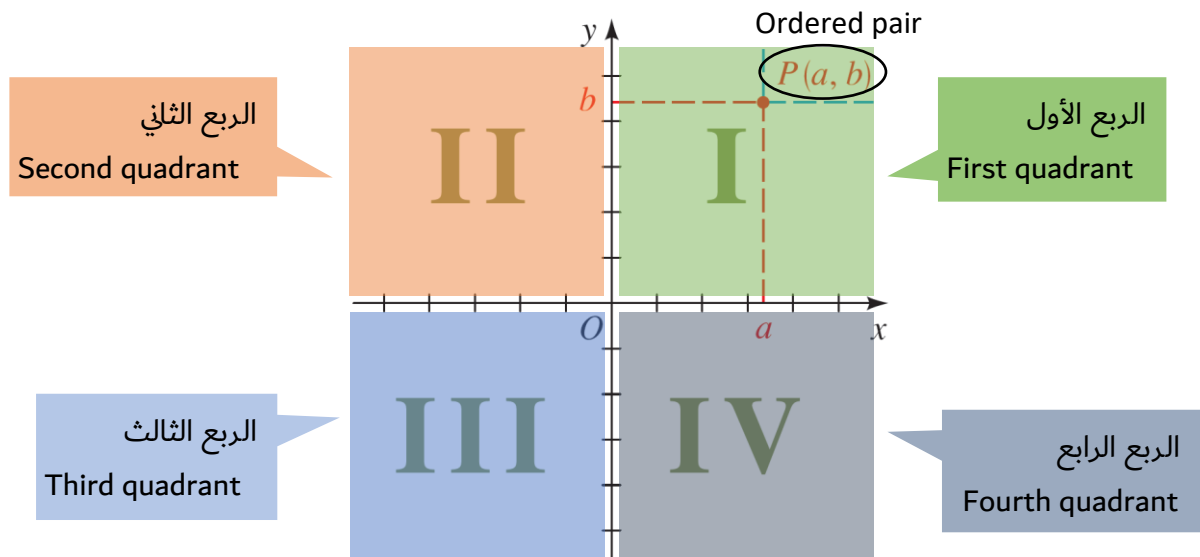


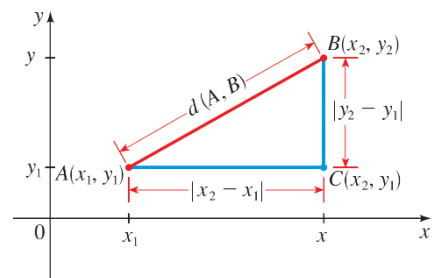
Section 1.9 – The coordinate plane; Graphs of equations; Circles

- The coordinate plane المستوى الإحداثي.



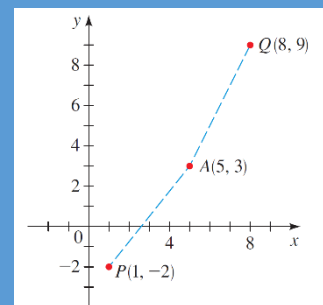
- The distance formula المسافة بين نقطتين

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 1

Which of the points $P(1, -2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$?



Solution

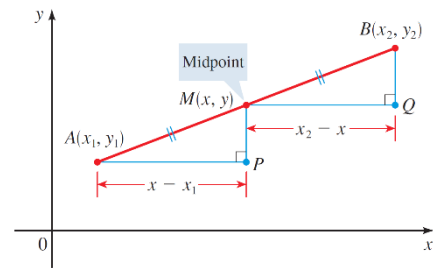
$$d(P, A) = \sqrt{(5-1)^2 + (3-(-2))^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$d(Q, A) = \sqrt{(5-8)^2 + (3-9)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{49}$$

point P is closer

- Midpoint formula منتصف المسافة بين نقطتين

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



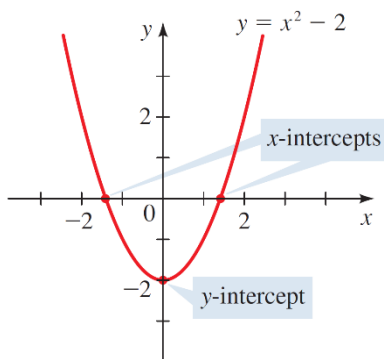
Example 2

Find the midpoint coordinates for the line segment from $P(1,2)$ to $Q(5,9)$.

Solution

$$M = \left(\frac{1+5}{2}, \frac{2+9}{2} \right)$$

$$= \left(3, \frac{11}{2} \right)$$



- عند رسم معادلة من متغيرين (x, y) قد تتقاطع الرسمة مع محور x أو محور y

لإيجاد **x-intercept** (نقاط التقاطع مع x):

نضع $y = 0$ ونوجد قيمة x

لإيجاد **y-intercept** (نقاط التقاطع مع y):

نضع $x = 0$ ونوجد قيمة y

Example 3

Find the x - and y -intercepts of the graph of the equation $y = x^2 - 2$.

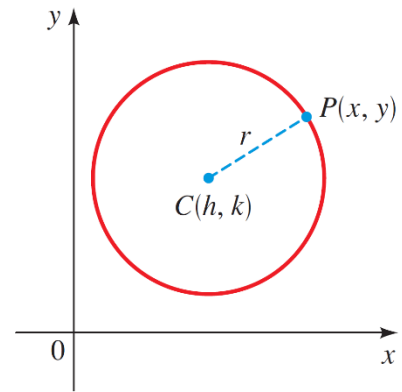
Solution

$$\begin{aligned} &\text{x-intercept} \\ &0 = x^2 - 2 \\ &2 = x^2 \\ &x = \pm\sqrt{2} \end{aligned}$$

$$\begin{aligned} &\text{y-intercept} \\ &y = 0 - 2 \\ &y = -2 \end{aligned}$$

- An equation of a circle دائرة with center مركز (h, k) and radius نصف قطر r is

$$(x - h)^2 + (y - k)^2 = r^2$$



Example 4

Find an equation of the circle with radius 3 and center $(2, -5)$.

Solution

$$(x - 2)^2 + (y + 5)^2 = 9$$

Example 5

Find an equation of the circle that has the points $P(1, 8)$ and $Q(5, -6)$ as the endpoints of a diameter.

Solution

$$M = \left(\frac{1+5}{2}, \frac{8-6}{2} \right) = (3, 1)$$

$$\begin{aligned} r = d(M, P) &= \sqrt{(3-1)^2 + (1-8)^2} \\ &= \sqrt{2^2 + (-7)^2} = \sqrt{53} \end{aligned}$$

$$(x - 3)^2 + (y - 1)^2 = 53$$

Example 6

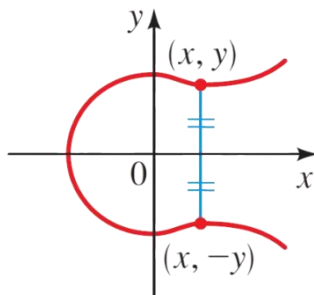
Show that the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ represents a circle, and find the center and radius of the circle.

Solution

$$\begin{aligned}
 x^2 + 2x + y^2 - 6y &= -7 \\
 x^2 + 2x + \left(\frac{2}{2}\right)^2 + y^2 - 6y + \left(\frac{-6}{2}\right)^2 &= -7 + \left(\frac{2}{2}\right)^2 + \left(\frac{-6}{2}\right)^2 \\
 x^2 + 2x + 1 + y^2 - 6y + 9 &= 3 \\
 (x+1)^2 + (y-3)^2 &= 3 \\
 \text{Center } (-1, 3) \quad r &= \sqrt{3}
 \end{aligned}$$

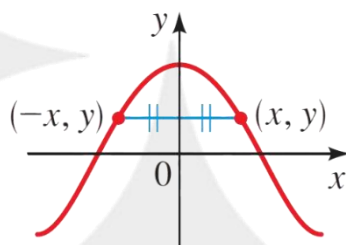
- Types of symmetry التماثل

1. With respect to x -axis:



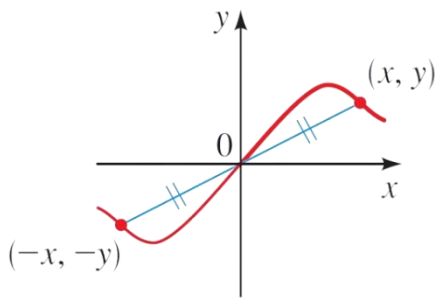
عند وضع $-y$ بدل y في المعادلة نحصل على نفس المعادلة الأصلية

2. With respect to y -axis:



عند وضع $-x$ بدل x في المعادلة نحصل على نفس المعادلة الأصلية

3. With respect to the origin:



عند وضع $-x$ بدل x و $-y$ بدل y في المعادلة نحصل على نفس المعادلة الأصلية

Example 7

Test the equation for symmetry.

(a) $x = y^2$

(b) $y = x^3 - 9x$

Solution

(a) ① $-x = y^2$

Not symmetric $[y]$

② $x = (-y)^2$
 $x = y^2$

symmetric $[x]$

③ $-x = (-y)^2$
 $-x = y^2$

Not symmetric $[origin]$

(b) ① $y = (-x)^3 - 9(-x)$

$y = -x^3 + 9x$

Not symmetric $[y]$

② $-y = x^3 - 9x$

Not symmetric $[x]$

③ $-y = -x^3 + 9x$
 $\neq y = -(x^3 - 9x)$
 $y = x^3 - 9x$

symmetric origin

Problems

- Which of the points $A(6, 7)$ or $B(-5, 8)$ is closer to the origin?

- Which of the points $C(-6, 3)$ or $D(3, 0)$ is closer to the point $E(-2, 1)$?

- If $M(6, 8)$ is the midpoint of the line segment AB and if A has coordinates $(2, 3)$, find the coordinates of B .

- Find the x - and y -intercepts of the graph of the equation.

(a) $y = x + 6$

(b) $y = x^2 - 5$

- Find the center and radius of the circle.

(a) $x^2 + y^2 = 9$

(b) $x^2 + (y - 4)^2 = 1$

- Find an equation of the circle that satisfies the given conditions.

(a) Center $(2, -1)$; radius 3

(b) Endpoints of a diameter are $P(-1, 1)$ and $Q(5, 9)$

- Show that the equation $x^2 + y^2 + 4x - 6y + 12 = 0$ represents a circle, and find the center and radius of the circle.

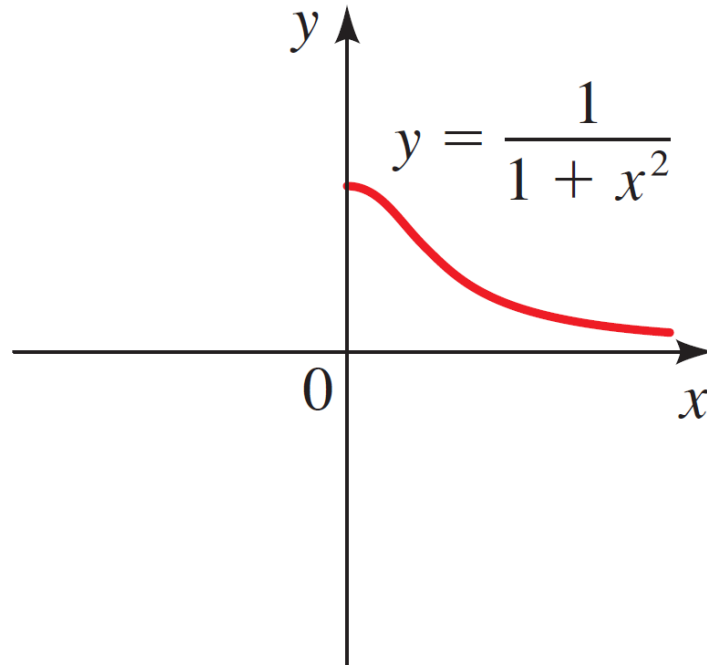
- Test the equation for symmetry.

(a) $y = x^4 + x^2$

(b) $x^2y^2 + xy = 1$

- Complete the graph using the given symmetry property:

(a) Symmetric with respect to the y -axis



(b) Symmetric with respect to the x -axis

