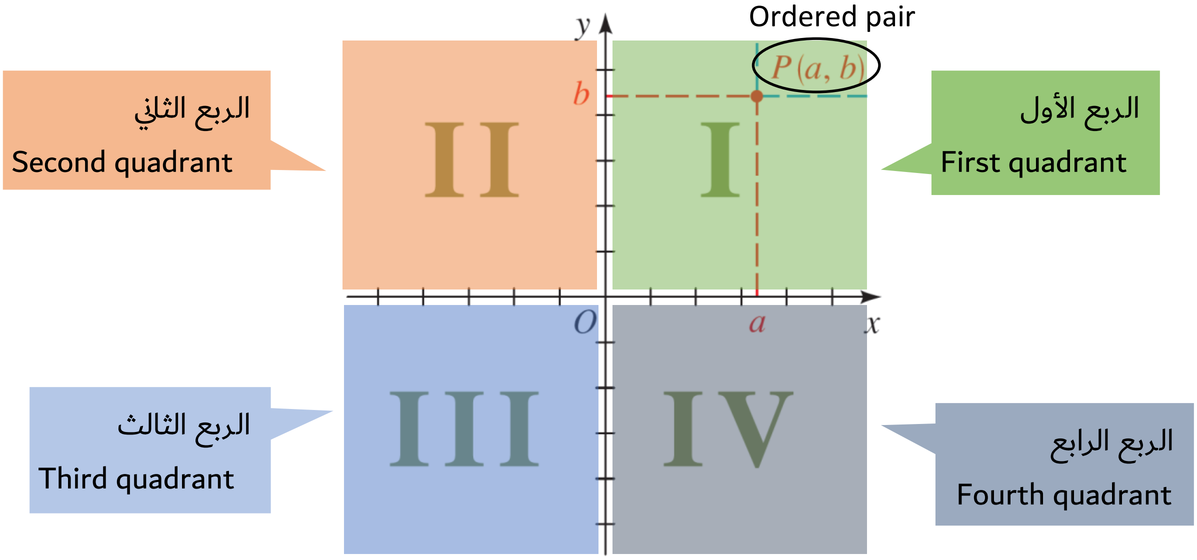
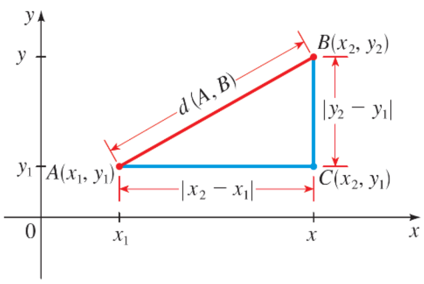
Section 1.9 – The coordinate plane; Graphs of equations; Circles

- The coordinate plane المستوى الإحداثي.



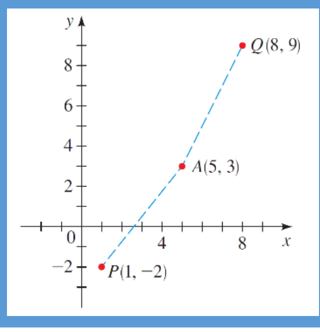
- The distance formula المسافة بين نقطتين

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 1

Which of the points $P(1,-2)$ or $Q(8,9)$ is closer to the point $A(5,3)$?



Solution

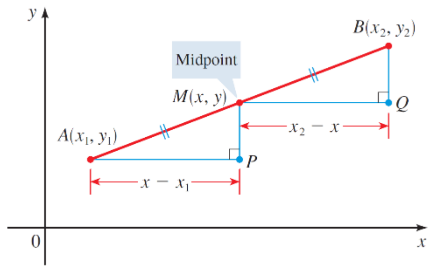
$d(P,A) = \sqrt{(5-1)^2 + (3-(-2))^2} = \sqrt{4^2+5^2} = \sqrt{41}$

$d(Q,A) = \sqrt{(5-8)^2 + (3-9)^2} = \sqrt{(-3)^2+(-6)^2} = \sqrt{49}$

$\boxed{\text{Point } P \text{ is closer}}$

- Midpoint formula منتصف المسافة بين نقطتين

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



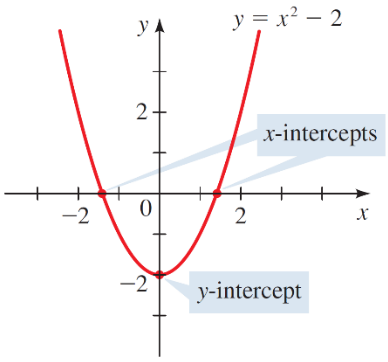
Example 2

Find the midpoint coordinates for the line segment from $P(1,2)$ to $Q(5,9)$.

Solution

$M = \left(\frac{1+5}{2}, \frac{2+9}{2}\right)$

$= \left(3, \frac{11}{2}\right)$



- عند رسم معادلة من متغيرين (x, y) قد تتقاطع الرسمة مع محور x أو محور y

لإيجاد x-intercept (نقاط التقاطع مع x):

نضع $y = 0$ ونوجد قيمة x

لإيجاد y-intercept (نقاط التقاطع مع y):

نضع $x = 0$ ونوجد قيمة y

Example 3

Find the x- and y-intercepts of the graph of the equation $y = x^2 - 2$.

Solution

x-intercept

$0 = x^2 - 2$

$2 = x^2$

$x = \pm\sqrt{2}$

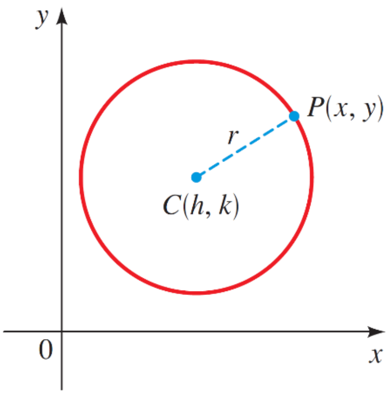
y-intercept

$y = 0 - 2$

$y = -2$

- An <u>equation of a circle</u> دائرة with center مركز (h, k) and radius نصف قطر r is

$$(x-h)^2+(y-k)^2=r^2$$



Example 4

Find an equation of the circle with radius 3 and center $(2, -5)$.

Solution

$$(x-2)^2+(y+5)^2=9$$

Example 5

Find an equation of the circle that has the points $P(1,8)$ and $Q(5,-6)$ as the endpoints of a diameter.

Solution

$$M=\left(\frac{1+5}{2}, \frac{8-6}{2}\right)=(3,1)$$

$$r=d(M,P)=\sqrt{(3-1)^2+(1-8)^2}$$

$$=\sqrt{2^2+(-7)^2}=\sqrt{53}$$

$$(x-3)^2+(y-1)^2=53$$

Example 6

Show that the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ represents a circle, and find the center and radius of the circle.

Solution

$$x^2 + 2x \quad + y^2 - 6y \quad = -7$$

$$x^2 + 2x + \left(\frac{2}{2}\right)^2 + y^2 - 6y + \left(\frac{-6}{2}\right)^2 = -7 + \left(\frac{2}{2}\right)^2 + \left(\frac{-6}{2}\right)^2$$

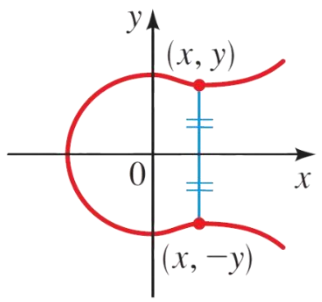
$$x^2 + 2x + 1 + y^2 - 6y + 9 = 3$$

$$(x+1)^2 + (y-3)^2 = 3$$

Center $(-1, 3)$ $\qquad r = \sqrt{3}$

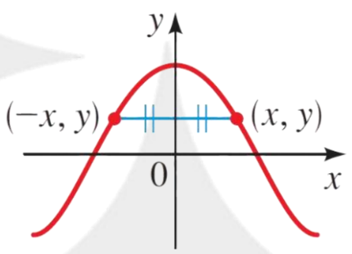
- Types of **symmetry التماثل**

1. With respect to x-axis:



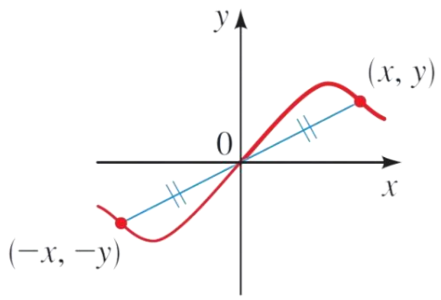
عند وضع $-y$ بدل y في المعادلة نحصل على نفس المعادلة الأصلية

2. With respect to y-axis:



عند وضع $-x$ بدل x في المعادلة نحصل على نفس المعادلة الأصلية

3. With respect to the origin:



عند وضع $-x$ بدل x و $-y$ بدل y في المعادلة نحصل على
نفس المعادلة الأصلية

Example 7

Test the equation for symmetry.

(a) $x = y^2$

(b) $y = x^3 - 9x$

Solution

(a) ① $-x = y^2$ Not symmetric $\boxed{y}$

② $x = (-y)^2$

$x = y^2$ symmetric $\boxed{x}$

③ $-x = (-y)^2$

$-x = y^2$ Not symmetric $\boxed{\text{origin}}$

(b) ① $y = (-x)^3 - 9(-x)$

$y = -x^3 + 9x$ Not symmetric $\boxed{y}$

② $-y = x^3 - 9x$ Not symmetric $\boxed{x}$

③ $-y = -x^3 + 9x$

$\cancel{-}y = \cancel{-}(x^3 - 9x)$

$y = x^3 - 9x$ symmetric origin

Problems

- Which of the points $A(6,7)$ or $B(-5,8)$ is closer to the origin?

$$d_{AO} = \sqrt{(6-0)^2 + (7-0)^2}$$

$$= \sqrt{36 + 49}$$

$$= \sqrt{85}$$

$$d_{BO} = \sqrt{(-5-0)^2 + (8-0)^2}$$

$$= \sqrt{25 + 64}$$

$$= \sqrt{89}$$

Point is closer $\quad d_A < d_B$

- Which of the points $C(-6,3)$ or $D(3,0)$ is closer to the point $E(-2,1)$?

$$d_{CE} = \sqrt{(-6-(-2))^2 + (3-1)^2}$$

$$= \sqrt{(-4)^2 + (2)^2}$$

$$= \sqrt{16 + 4} = \sqrt{20}$$

$$d_{DE} = \sqrt{(3-(-2))^2 + (0-1)^2}$$

$$= \sqrt{5^2 + (-1)^2}$$

$$= \sqrt{25 + 1} = \sqrt{26}$$

Point C is closer

- If $M(6,8)$ is the midpoint of the line segment AB and if A has coordinates $(2,3)$, find the coordinates of B.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$\therefore$

$$\frac{2 + x_2}{2} = 6 \qquad \frac{3 + y_2}{2} = 8$$

$$2 + x_2 = 6 \cdot 2 \qquad 3 + y_2 = 8 \cdot 2$$

$$x_2 = 12 - 2 \qquad y_2 = 16 - 3$$

$$x_2 = 10 \qquad y_2 = 13$$

$B(10, 13)$

- Find the x- and y-intercepts of the graph of the equation.

(a) $y = x + 6$

x-intercept

$0 = x + 6$

$x = -6$

y-intercept

$y = 0 + 6$

$y = 6$

(b) $y = x^2 - 5$

x-intercept

$0 = x^2 - 5$

$x^2 = 5$

$x = \pm\sqrt{5}$

y-intercept

$y = 0 - 5$

$y = -5$

- **Find the center and radius of the circle.**

(a) $x^2 + y^2 = 9$

Center: $(0,0)$

radius $= \sqrt{9} = 3$

(b) $x^2 + (y-4)^2 = 1$

Center : $(0, 4)$

radius $= \sqrt{1} = 1$

- Find an equation of the circle that satisfies the given conditions.

(a) Center $(2,-1)$; radius 3

$(x-2)^2+(y+1)^2=9$

(b) Endpoints of a diameter are $P(-1,1)$ and $Q(5,9)$

Center : $\left(\frac{-1+5}{2}, \frac{1+9}{2}\right)$

$=\left(\frac{4}{2}, \frac{10}{2}\right)=(2,5)$

radius $= d_{PC}=\sqrt{(-1-2)^2+(1-5)^2}$

$=\sqrt{(-3)^2+(-4)^2}$

$=\sqrt{9+16}$

$=\sqrt{25}=5$

$(x-2)^2+(y-5)^2=25$

- Show that the equation $x^2 + y^2 + 4x - 6y + 12 = 0$ represents a circle, and find the center and radius of the circle.

$x^2 + 4x \quad + y^2 - 6y \quad = -12$

$x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 6y + \left(\frac{-6}{2}\right)^2 = -12 + \left(\frac{4}{2}\right)^2 + \left(\frac{-6}{2}\right)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = -12 + 4 + 9$

$(x+2)^2 + (y-3)^2 = 1$

Center: $(-2, 3)$

radius $= \sqrt{1} = 1$

- Test the equation for symmetry.

(a) $y = x^4 + x^2$

$-y = x^4 + x^2$ Not symmetric about x

$y = (-x)^4 + (-x)^2$

$y = x^4 + x^2$ symmetric about y

$-y = (-x)^4 + (-x)^2$

$-y = x^4 + x^2$ Not symmetric about the origin

(b) $x^2y^2 + xy = 1$

$x^2(-y)^2 + x(-y) = 1$

$x^2y^2 - xy = 1$ Not symmetric about x

$(-x)^2y^2 + (-x)y = 1$

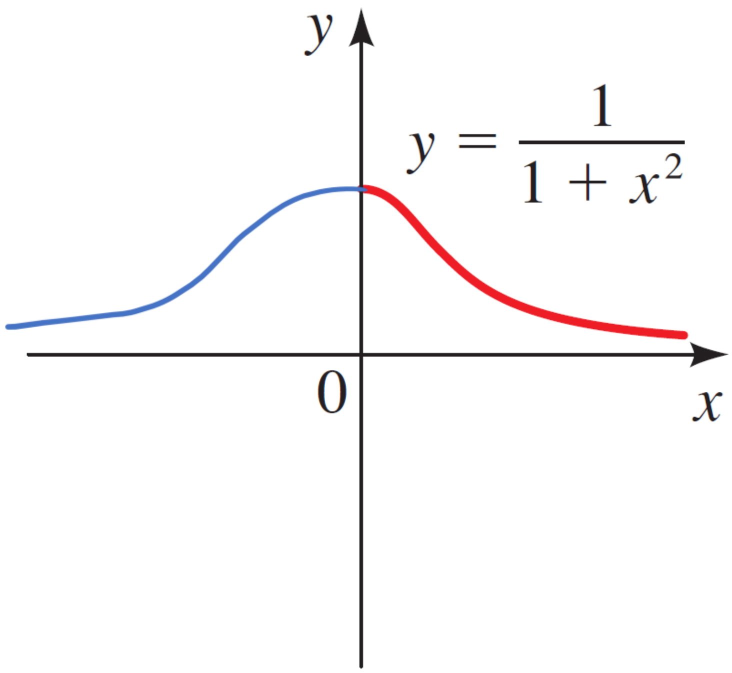
$x^2y^2 - xy = 1$ Not symmetric about y

$(-x)^2(-y)^2 + (-x)(-y) = 1$

$x^2y^2 + xy = 1$ symmetric about the origin

- Complete the graph using the given symmetry property:

(a) Symmetric with respect to the y-axis



(b) Symmetric with respect to the x-axis