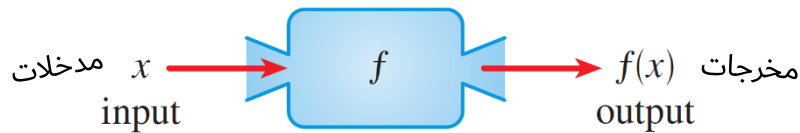


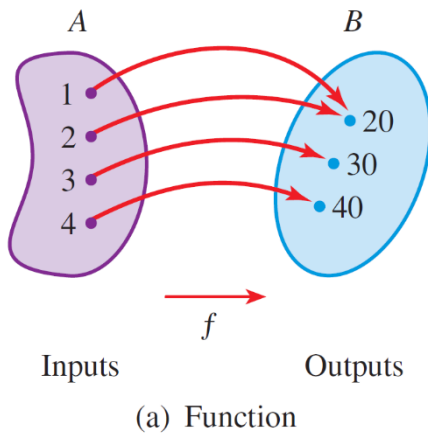
Section 2.1 – Functions

- A **function** الدالة f is a rule that assigns to each element " x " in a set A exactly one element " $f(x)$ " in a set B

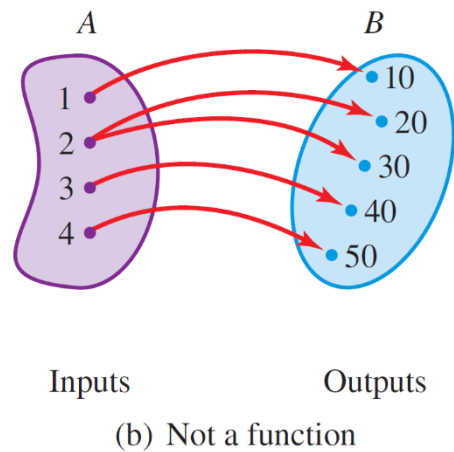
وصف علاقة بين عناصر مجموعة A (كل عنصر منها يسمى x)، و عناصر مجموعة B (كل عنصر يسمى $f(x)$)



الدالة كأنها ماكينة: تدخل لها قيمة، تطلع منها قيمة أخرى.



دالة لأن كل عنصر في مجموعة المدخلات A يقابله قيمة واحدة فقط من المجموعة B .



ليست دالة لأن أحد عناصر المجموعة A يقابله قيمتين من المجموعة B .

لاحظ

مجموعة المدخلات تسمى: input أو domain ومعناها بالعربي: مجال الدالة
مجموعة المخرجات تسمى: output أو range ومعناها بالعربي: مدى الدالة

Example 1

A function f is defined by the formula

$$f(x) = x^2 + 4$$

(a) Evaluate $f(3)$, $f(-2)$, and $f(\sqrt{5})$

(b) Find the domain and range of f .

Solution

$$\begin{aligned} \text{(a)} \quad f(3) &= 3^2 + 4 \\ &= 9 + 4 = 13 \end{aligned}$$

$$f(-2) = (-2)^2 + 4 = 8$$

$$\begin{aligned} f(\sqrt{5}) &= (\sqrt{5})^2 + 4 \\ &= 5 + 4 = 9 \end{aligned}$$

$$\text{(b)} \quad D: \mathbb{R} \quad \begin{array}{l} \text{جميع الأعداد الحقيقية} \\ \text{واجابة أخرى } (-\infty, \infty) \end{array}$$

$$\text{Range: } \because x^2 \geq 0$$

$$\therefore \text{Range: } [4, \infty)$$

Example 2

Let $f(x) = 3x^2 + x - 5$. Evaluate each function value.

- (a) $f(-2)$ (b) $f(0)$ (c) $f(4)$ (d) $f\left(\frac{1}{2}\right)$

Solution

$$\begin{aligned} \text{(a)} \quad f(-2) &= 3(-2)^2 + (-2) - 5 \\ &= 3 \cdot 4 - 2 - 5 = 12 - 2 - 5 = 5 \end{aligned}$$

$$\text{(b)} \quad f(0) = 3(0)^2 + 0 - 5 = -5$$

$$\begin{aligned} \text{(c)} \quad f(4) &= 3(4)^2 + 4 - 5 \\ &= 3 \cdot 16 + 4 - 5 = 48 + 4 - 5 = 47 \end{aligned}$$

$$\text{(d)} \quad f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 5 = -\frac{15}{4}$$

Example 3

Given that:

$$C(x) = \begin{cases} 39 & \text{if } 0 \leq x \leq 2 \\ 39 + 15(x - 2) & \text{if } x > 2 \end{cases}$$

Find $C(0.2)$, $C(2)$, and $C(4)$

Solution

$$C(0.2) = 39$$

$$C(2) = 39$$

$$\begin{aligned} C(4) &= 39 + 15(4 - 2) \\ &= 39 + 15 \cdot 2 = 69 \end{aligned}$$

Example 4

Find the domain of each function:

(a) $f(x) = \frac{1}{x^2 - x}$

(b) $g(x) = \sqrt{9 - x^2}$

(c) $h(t) = \frac{t}{\sqrt{t+1}}$

(d) $f(x) = \sqrt[3]{x - 2}$

Solution

$$\begin{aligned} \text{(a)} \quad x^2 - x &= 0 \\ x(x - 1) &= 0 \\ x = 0 \quad x &= 1 \end{aligned}$$

$$D: \{x \mid x \neq 0 \text{ and } x \neq 1\}$$

or in interval notation

$$D: (-\infty, 0) \cup (0, 1) \cup (1, \infty)$$

$$\begin{aligned} \text{(b)} \quad 9 - x^2 &\geq 0 \\ -x^2 &\geq -9 \\ x^2 &\leq 9 \\ -3 &\leq x \leq 3 \end{aligned}$$

$$D: [-3, 3]$$

$$\begin{aligned} \text{(c)} \quad t + 1 &> 0 \\ t &> -1 \end{aligned}$$

$$D: (-1, \infty) = \{t \mid t > -1\}$$

Problems

- Evaluate the function at the indicated values:

$$f(x) = x^3 + 2x - 1;$$

(a) $f(-2)$ (b) $f(a)$ (c) $f(-a)$ (d) $f\left(\frac{1}{2}\right), \frac{f(a+h)-f(h)}{h}, h \neq 0$

- Evaluate the function at the indicated values:

$$f(x) = \frac{1-2x}{3};$$

(a) $f(-2)$ (b) $f(a - 1)$ (c) $f(a)$

- Evaluate the piecewise defined function at the indicated values:

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$$

(a) $f(-2)$ (b) $f(-1)$ (c) $f(0)$ (d) $f(1)$ (e) $f(2)$

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ x + 1 & \text{if } 0 \leq x \leq 2 \\ x - 1 & \text{if } x > 2 \end{cases}$$

(a) $f(-5)$ (b) $f(0)$ (c) $f(1)$ (d) $f(2)$ (e) $f(5)$

- Use the function to evaluate the indicated expressions and simplify

$$f(x) = 3x - 1;$$

(a) $f(2x)$ (b) $2f(x)$ (c) $f(x + 2)$ (d) $f(x^2)$

- Find the domain of the function

$$f(x) = \frac{1}{x-3}$$

$$g(x) = \sqrt{x^2 - 4}$$

$$g(x) = \sqrt{x^2 - 2x - 8}$$

$$g(x) = \frac{3}{\sqrt{x-4}}$$