

Section 2.7 – Combining Functions

- Two functions can be added, subtracted, multiplied, or divided

يمكن جمع، طرح، ضرب، و قسمة الدوال. و ينتج عنها دالة جديدة مداها domain هو تقاطع intersection

مدى الدالتين $A \cap B$

Example 1

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

(a) Find the functions $f + g$, $f - g$, fg , and f/g and their domains.

(b) Find $(f + g)(4)$, $(f - g)(4)$, $(fg)(4)$, and $f/g(4)$

Solution

$$f(x) = \frac{1}{x-2}$$

$$D: (-\infty, 2) \cup (2, \infty)$$

$$g(x) = \sqrt{x}$$

$$D: [0, \infty)$$

$$(a) f + g = \frac{1}{x-2} + \sqrt{x}$$

$$D: D(f(x)) \cap D(g(x))$$

$$D: [0, 2) \cup (2, \infty) = \{x \mid x \geq 0, x \neq 2\}$$

$$f - g = \frac{1}{x-2} - \sqrt{x} \quad D: [0, 2) \cup (2, \infty)$$

$$fg = \frac{\sqrt{x}}{x-2} \quad D: [0, 2) \cup (2, \infty)$$

$$f/g = \frac{1}{x-2} \div \sqrt{x} = \frac{1}{\sqrt{x}(x-2)}$$

$$D: [0, 2) \cup (2, \infty)$$

$$(b) f+g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{1}{2} + 2 = \frac{5}{2}$$

- For the two functions f and g , the **composite function** $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x))$$

الدالة المركبة تتكون من دالتين بحيث تستخدم مخرجات واحدة كمدخلات للدالة الأخرى

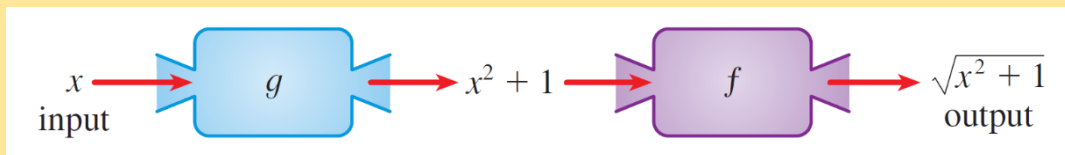
Example:

$$f(x) = \sqrt{x} \text{ and } g(x) = x^2 + 1$$

$$f \circ g = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

لاحظ

بدأنا من الدالة الداخلية (في المثال $g(x)$) ثم نستخدم الناتج منها كمدخل للدالة الخارجية $f(x)$



Example 2

Let $f(x) = x^2$ and $g(x) = x - 3$

(a) Find the functions $f \circ g$, and $g \circ f$ and their domains.

(b) Find $(f \circ g)(5)$, and $(g \circ f)(7)$

Solution

$$(a) \quad f \circ g = f(g(x)) = (x-3)^2$$

$$D: \mathbb{R}$$

$$g \circ f = g(f(x)) = x^2 - 3$$

$$D: \mathbb{R}$$

$$(b) \quad (f \circ g)(5) = (5-3)^2 = 4$$

$$(g \circ f)(7) = 7^2 - 3 = 46$$

Example 3

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find the following functions and their domains

(a) $f \circ g$

(b) $g \circ f$

(c) $f \circ f$

(d) $g \circ g$

Solution

$$(a) f \circ g = f(g(x))$$

$$= f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$$

$$2-x \geq 0$$

$$-x \geq -2$$

$$x \leq 2$$

$$D = (-\infty, 2]$$

$$(b) g \circ f = g(f(x))$$

$$= g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$$

$$2-\sqrt{x} \geq 0 \quad \text{and} \quad x \geq 0$$

$$-\sqrt{x} \geq -2$$

$$\sqrt{x} \leq 2$$

$$x \leq 4$$

$$D: [0, 4]$$

$$(c) f \circ f = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}}$$

$$D: [0, \infty) = \{x \mid x \geq 0\}$$

Problems

- Find $f + g$, $f - g$, fg , and f/g and their domains.

(a) $f(x) = x^2 + x$, $g(x) = x^2$

(b) $f(x) = 5 - x$, $g(x) = x^2 - 3x$

- Use $f(x) = 2x - 3$ and $g(x) = 4 - x^2$ to evaluate the expression.

(a) $f(g(0))$

(b) $g(f(0))$

- Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$$f(x) = x^2, g(x) = x + 1$$

- Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$$f(x) = \frac{x}{x+1}, g(x) = 2x - 1$$

- Use the table to evaluate the expression.

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

(a) $f(g(2))$

(b) $(f \circ g)(6)$

(c) $(f \circ f)(5)$

(d) $f(5) - 3g(6)$

- Use the given graphs of f and g to evaluate the expression.

(a) $f(g(2))$

(b) $(g \circ f)(4)$

(c) $(g \circ g)(-2)$

