Section 3.3 – Dividing Polynomials

- If $P(x)$ and $D(x)$ are polynomials دوال كثيرة حدود, and $D(x) \neq 0$, then we can divide نقسم $P(x)$ by $D(x)$ such that:

Dividend, Quotient, Remainder, Divisor

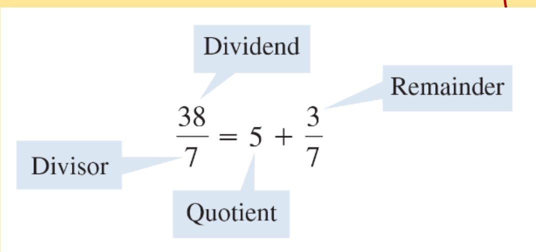
$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)} \Longrightarrow P(x) = D(x) \cdot Q(x) + R(x)$$

يمكن قسمة دالة كثيرة حدود على دالة كثيرة حدود بنفس طريق قسمة رقم على رقم، و تسمى هذه الطريقة:

long division القسمة المطولة

لاحظ

طريقة قسمة الدوال نفس الأرقام



Example 1

Divide $6x^2 - 26x + 12$ by $x - 4$

Solution

$$\begin{array}{r} 6x \quad - 2 \\ x-4 \,\big)\, \overline{6x^2 - 26x + 12} \\ -(6x^2 - 24x) \downarrow \\ \hline -2x + 12 \\ -(-2x + 8) \\ \hline 4 \end{array}$$

$$\frac{6x^2 - 26x + 12}{x - 4} = 6x - 2 + \frac{4}{x-4}$$

Example 2

Let $P(x) = 8x^4 + 6x^2 - 3x + 1$ and $D(x) = 2x^2 - x + 2$. Divide $P(x)$ by $D(x)$.

Solution

$$
\begin{array}{r|l}
 & 4x^2 + 2x \\
2x^2 - x + 2 & 8x^4 \qquad + 6x^2 - 3x + 1 \\
 & -\left(8x^4 - 4x^3 + 8x^2\right) \\
 \hline
 & 4x^3 - 2x^2 - 3x + 1 \\
 & -\left(4x^3 - 2x^2 + 4x\right) \\
 \hline
 & -7x + 1
\end{array}
$$

$$\frac{8x^4 + 6x^2 - 3x + 1}{2x^2 - x + 2} = 4x^2 + 2x + \frac{-7x + 1}{2x^2 - x + 2}$$

- Synthetic division القسمة التركيبية a quick method of dividing polynomials, it can be used when the divisor المقسوم عليه is of the form $x - c$

Example 3

Use synthetic division to divide $2x^3 - 7x^2 + 5$ by $x - 3$

Solution

Synthetic division

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & \downarrow & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & -4 \end{array}$$

$\times$

Long division

$$\begin{array}{r} 2x^2 - x - 3 \\ x-3 \overline{\smash{)}\, 2x^3 - 7x^2 \quad + 5} \\ \underline{\mp 2x^3 \pm 6x^2 \qquad\ \downarrow} \\ -x^2 \quad + 5 \\ \underline{\pm x^2 \mp 3x \ \downarrow} \\ -3x + 5 \\ \underline{\pm 3x \mp 9} \\ -4 \end{array}$$

$$\frac{2x^3 - 7x^2 + 5}{x - 3} = 2x^2 - x - 3 - \frac{4}{x - 3}$$

- **Remainder Theorem:** If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$

Example 4

Find the remainder of dividing $P(x) = x^3 + 3x^2 - 7x + 6$ by $x - 2$

Solution

$$\text{remainder} = P(2) = 2^3 + 3(2)^2 - 7\cdot 2 + 6$$

$$= 8 + 12 - 14 + 6$$

$$= 12$$

Another sol.

$$\begin{array}{r|l}
 & x^2 + 5x + 3 \\
\hline
x - 2 & x^3 + 3x^2 - 7x + 6 \\
 & -x^3 \overset{+}{-} 2x^2 \quad \downarrow \quad \downarrow \\
\hline
 & 5x^2 - 7x + 6 \\
 & -5x^2 \overset{+}{-} 10x \\
\hline
 & 3x + 6 \\
 & -3x \overset{+}{-} 6 \\
\hline
 & 12
\end{array}$$

Example 5

Find the remainder of dividing $P(x) = x^3 + 3x^2 - 7x + 6$ by $x + 2$

Solution

$$P(-2) = (-2)^3 + 3(-2)^2 - 7(-2) + 6$$

$$= -8 + 12 + 14 + 6$$

$$= 24$$

- **Factor Theorem:** c is a zero of P if, and only if, $x - c$ is a factor of $P(x)$

$x - c$ هي أحد معاملات الدالة $P(x)$ (c هي أحد أصفار الدالة) إذا كانت $P(x)$ تقبل القسمة على $x - c$

بدون متبقي remainder

يعني $P(c) = 0$

Example 6

Which of the following is a factor of $P(x) = x^3 - 7x - 6$?

(a) $x + 2$ (b) $x - 1$ (c) $x + 4$ (d) $x + 3$

Solution

$\boxed{(a)}$ $P(-2) = (-2)^3 - 7(-2) - 6 = -8 + 14 - 6 = 0$

(b) $P(1) = 1^3 - 7 \cdot 1 - 6 = -12$

(c) $P(-4) = (-4)^3 - 7(-4) - 6 = -42$

(d) $P(-3) = (-3)^3 - 7(-3) - 6 = -12$

Example 7

Let $P(x) = x^3 - 7x + 6$. Show that 1 is a zero of $P(x)$, and use this fact to find all other real zeros.

Solution

$P(1) = 1^3 - 7(1) + 6 = 0$

$\therefore$ 1 is a zero of $P(x)$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$P(x) = (x-1)(x^2 + x - 6)$

$= (x-1)(x+3)(x-2)$

Zeros of $P(x)$ are: $\{1, -3, 2\}$

Problems

- Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$

(a) $P(x) = -x^3 - 2x + 6,\ D(x) = x + 1$

(b) $P(x) = 8x^4 + 4x^3 + 6x^2,\ D(x) = 2x^2 + 1$

- Find the quotient and remainder using long division

$$\frac{x^3 + 2x + 1}{x^2 - x + 3}$$

- Find the quotient and remainder using synthetic division

$$\frac{x^3 - 8x + 2}{x + 3}$$

- Use synthetic division and the Remainder Theorem to evaluate $P(c)$

$P(x) = 4x^2 + 12x + 5, c = -1$

- Use the Factor Theorem to show that $x - c$ is a factor of $P(x)$ for the given value of c.

$\boldsymbol{P(x) = x^3 - 3x^2 + 3x - 1,\ c = 1}$

-Show that the given value of c are zeros of $P(x)$, and find all other zeros of $P(x)$

$P(x) = x^3 + 2x^2 - 9x - 18,\ c = -2$