Section 2.1

Q1. If $f(x) = 3x - 5$. Find $\frac{f(x)-f(a)}{x-a}$ and simplify

Fall 24-25

$f(a) = 3a - 5$

$$\frac{f(x)-f(a)}{x-a} = \frac{3x-5-(3a-5)}{x-a}$$

$$= \frac{3x-\cancel{5}-3a+\cancel{5}}{x-a}$$

$$= \frac{3x-3a}{x-a}$$

$$= \frac{3\cancel{(x-a)}}{\cancel{x-a}} = 3$$

Q2. Consider the piecewise defined function

Summer 22-23

$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } -2 \le x \le 2 \\ x^2 + x + 1 & \text{if } x > 3 \end{cases}$$

Evaluate $2f(0) + f(4) - 7$

$2f(0) = 2\sqrt{9-0} = 2\sqrt{9} = 2(3) = 6$

$f(4) = 4^2 + 4 + 1 = 16 + 4 + 1 = 21$

$2f(0) + f(4) - 7 = 6 + 21 - 7 = 20$

Q3. Find the domain of $f(x) = \frac{\sqrt{36-x^2}}{x-6}$ **Fall 22-23**

$36 - x^2 \geq 0$

$-x^2 \geq -36$

$x^2 \leq 36$

$-6 \leq x \leq 6$

$[-6, 6)$

$x - 6 = 0$

$x = 6$

$\mathbb{R}/\{6\}$

$D: [-6, 6] \cap \mathbb{R}/\{6\}$

$D: [-6, 6)$

Q4. Find the domain of $f(x) = \sqrt{x-1} - \frac{1}{\sqrt{2-x}}$ **Spring 22-23**

$x - 1 \geq 0$

$x \geq 1$

$[1, \infty)$

$2 - x > 0$

$-x > -2$

$x < 2$

$(-\infty, 2)$

$>$ بدون يساوي لأنها في المقام

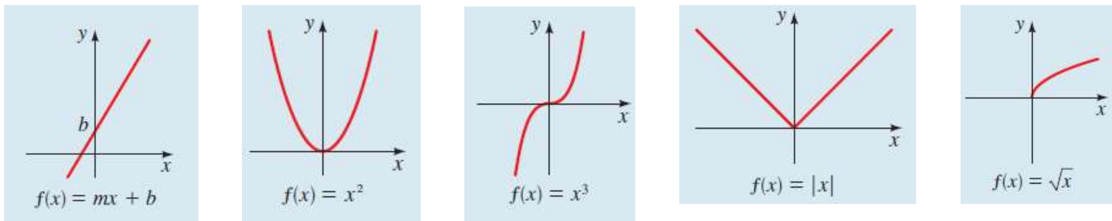
$D: (-\infty, 2) \cap [1, \infty)$

$D: [1, 2)$

Q5. Find the domain $f(x) = 5x^2 + 4,$ $0 \leq x \leq 2$

$D: [0, 2]$

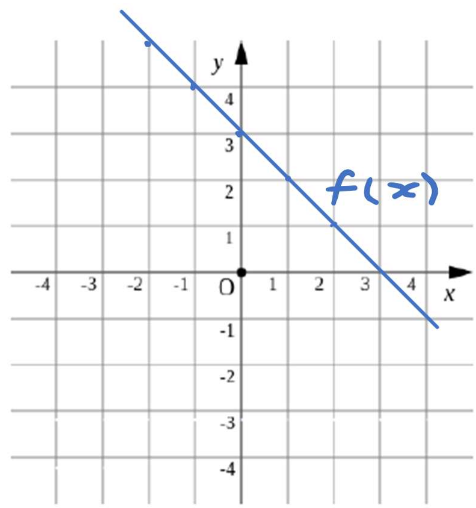
Section 2.2



Q1. Sketch graphs of the following functions

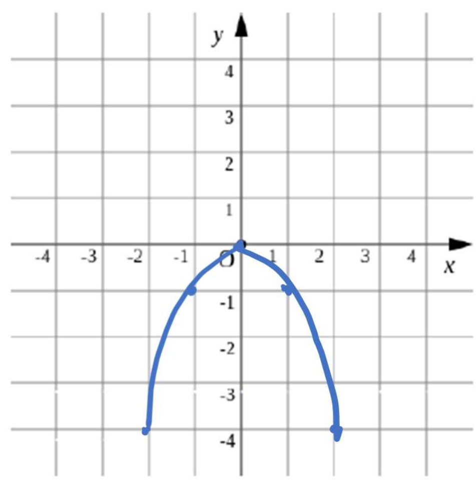
a) $f(x) = -x + 3$

x	$y = f(x)$
-2	$-(-2)+3 = 5$
-1	$-(-1)+3 = 4$
0	$-(0)+3 = 3$
1	$-(1)+3 = 2$
2	$-(2)+3 = 1$



b) $h(x) = -x^2$

x	$y = f(x)$
-2	$-(-2)^2 = -4$
-1	$-(-1)^2 = -1$
0	0
1	$-(1)^2 = -1$
2	$-(2)^2 = -4$



Q2. Determine whether the equation defines y as a function of x **Summer 24-25**

$$x^2 - y - 6 = 1$$

$$-y - 6 = -x^2 + 1$$
$$-y = -x^2 + 1 + 6$$
$$-y = -x^2 + 7$$
$$y = x^2 - 7$$

Function, any value of x gives one value of y

Q3. Determine whether the equation $\sqrt{x} - y^2 = 5$ defines y as a function of x. **Spring 22-23**

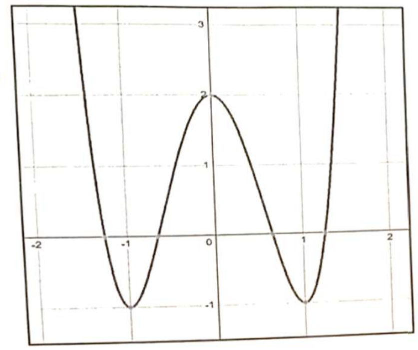
$$-y^2 = -\sqrt{x} + 5$$
$$y^2 = \sqrt{x} - 5$$
$$y = \pm\sqrt{\sqrt{x} - 5}$$

Not a function, any value of x gives two values of y ($\pm$)

Section 2.3

Q1. Given the graph of function f, find:

Fall 24-25



1. a) Local maximum 2

b) The value(s) of x at which the local maximum occurs 0

2. a) Local minimum -1

b) The value(s) of x at which the local minimum occurs

$-1, 1$

3. a) Domain of $f(x)$ $(-\infty, \infty)$

b) Range of $f(x)$ $[-1, \infty)$

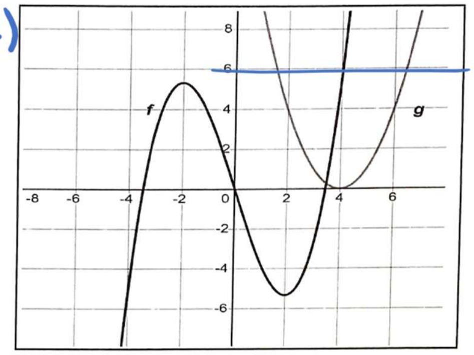
4. a) Intervals for $f(x)$ increasing $(-1, 0) \cup (1, \infty)$

b) Intervals for $f(x)$ decreasing $(-\infty, -1) \cup (0, 1)$

5. a) Find $f^{-1}(2)$ $= 0, -1.5, 1.5$

b) Find $f(-1)$ $= -1$

Q2. The graphs of f and g are given **Spring 22-23**



a) Find the interval on which f is decreasing $(-2,2)$

b) Find the local minimum value of g 0

c) Find the value of x at which f has a local maximum

-2

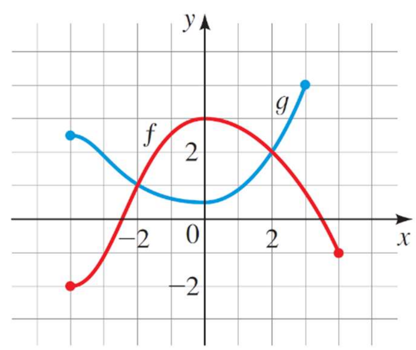
d) Determine whether f is even, odd or neither

Odd, symmetric about the origin

e) Is g one-to-one?

No, by Horizontal Line Test

Q3. Graphs of the functions f and g are given.



(a) Which is larger, $f(0)$ or $g(0)$?

$f(0)$

(b) Which is larger, $f(-3)$ or $g(-3)$?

$g(-3)$

(c) For which values of x is $f(x) = g(x)$?

$-2, 2$

(d) Find the values of x for which $f(x) \leq g(x)$.

$[-4,-2] \cup [2,3]$

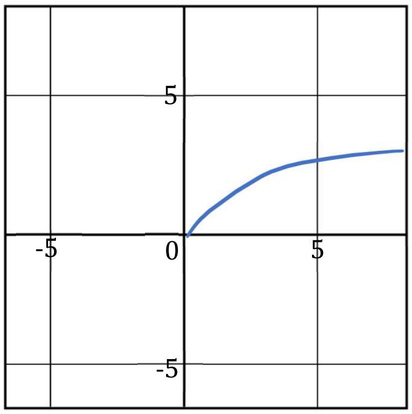
(e) Find the values of x for which $f(x) > g(x)$.

$(-2,2)$

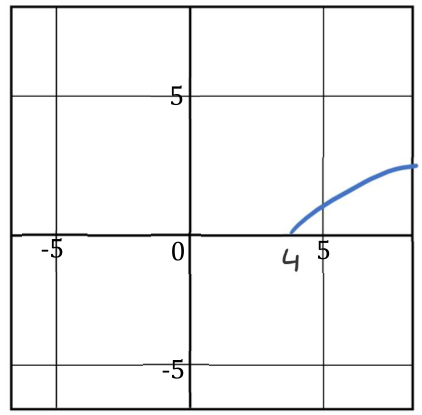
Section 2.6

Q1. Sketch the graph of each of the following functions: **Fall 24-25**

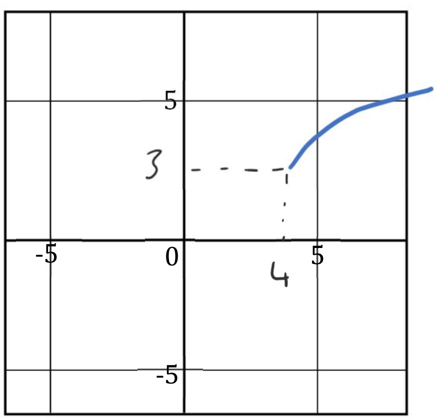
a. $f(x) = \sqrt{x}$



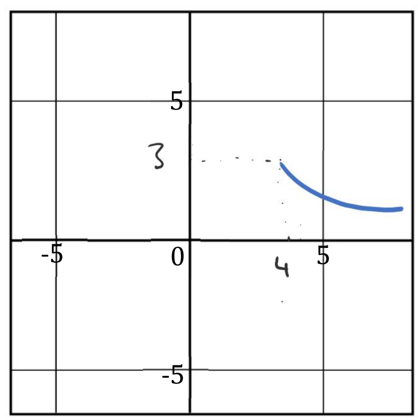
b. $f(x) = \sqrt{x-4}$



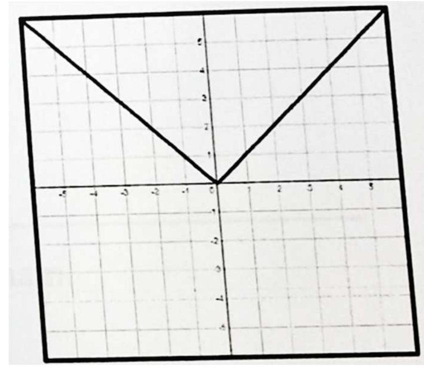
c. $f(x) = \sqrt{x-4} + 3$



d. $f(x) = -\sqrt{x-4} + 3$



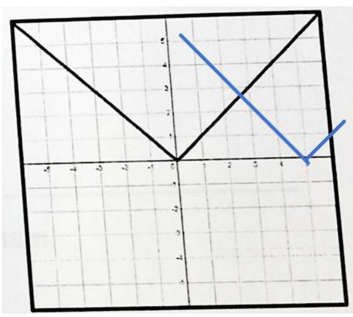
Q2. The graph of $f(x) = |x|$ is given on the right. Sketch the graph of each of the following functions: **Fall 22-23**

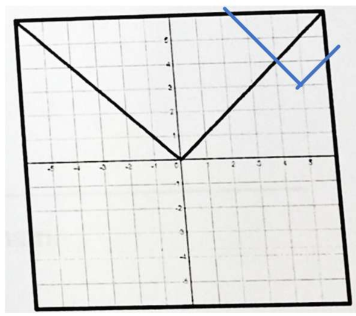


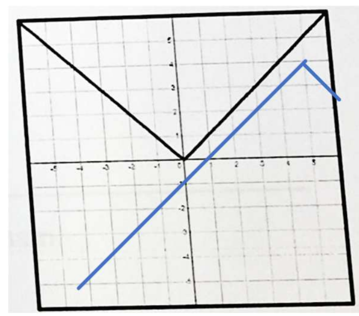
a. $f(x) = |x - 4|$

b. $f(x) = |x - 4| + 3$

c. $f(x) = -|x - 4| + 3$







Q3. Determine whether the function $f(x) = x^3 - x$ is even, odd, or neither. **Fall 23-24**

$$f(-x) = (-x)^3 - (-x)$$
$$= -x^3 + x$$
$$= -(x^3 - x)$$
$$= -f(x)$$

The function is odd

Section 2.7

Q1. Let $f(x) = x - 2$, $g(x) = x^2 - 4$. Evaluate the following: **Summer 24-25**

a. $\left(\frac{f}{g}\right)(3)$

$$\frac{f}{g} = \frac{x-2}{x^2-4}$$

$$\left(\frac{f}{g}\right)(3) = \frac{3-2}{3^2-4} = \frac{1}{9-4} = \frac{1}{5}$$

b. $(f \cdot g)(1)$

$$f \cdot g = (x-2)(x^2-4)\sqrt{x^2+1}$$

$$f \cdot g(1) = (1-2)(1^2-4) = (-1)(-3) = 3$$

Q2. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+4}$, find **Fall 23-24**

i. $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x))$$

$$= \left(\sqrt{x+4}\right)^2 - 1 = x + 4 - 1 = x + 3$$

ii. $g(f(1))$

$$f(1) = 1^2 - 1 = 0$$

$$g(f(1)) = g(0) = \sqrt{0+4} = \sqrt{4} = 2$$

Q3. Evaluate the following using the table. **Summer 24-25**

x	-1	1	2	3	4
$f(x)$	0	2	3	4	5
$g(x)$	2	2	5	10	17

a. $f^{-1}(0)$ $= -1$

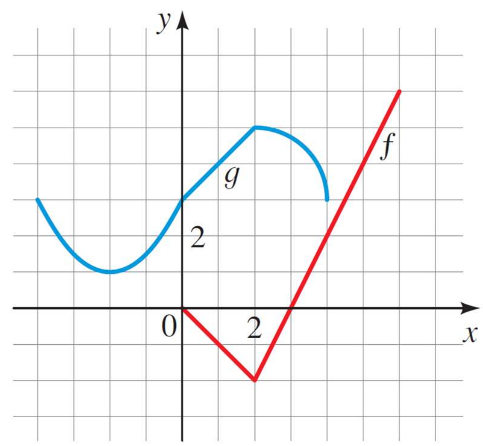
b. $(g \circ f)(3)$ $= g(4) = 17$

c. $(f \circ g)(1)$ $= f(2) = 3$

d. $2f(2) - g(-1)$ $= 2(3) - 2 = 6 - 2 = 4$

e. $f(f^{-1}(4))$ $= 4$

Q4. Use the given graphs of f and g to evaluate the expression.



(a) $f(g(2))$ $= f(5) = 4$

(b) $(g \circ f)(4)$ $= g(2) = 5$

(c) $(g \circ g)(-2)$ $= g(1) = 4$

Section 2.8

Q1. Determine whether the function $f(x) = -2x + 4$ is one-to-one

Fall 23-24

$f(a) = f(b)$

$-2a + 4 = -2b + 4 \qquad -4$

$-2a = -2b \qquad \div -2$

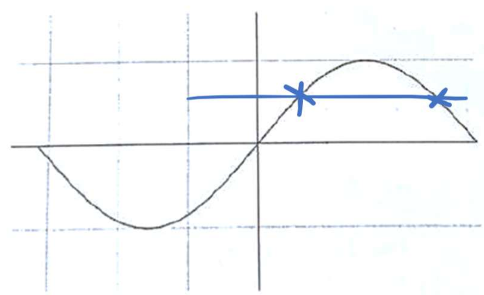
$a = b$

The function is one-to-one

Q2. From the following graph

Fall 24-25

a) Determine whether the graph represents the graph of a function



By Vertical Line Test the graph represents a function

b) Determine whether $f(x)$ is one-to-one or not

By Horizontal Line Test, the function is NOT one-to-one

Q3. Find the inverse of the function $f(x) = \frac{x+1}{x-2}$. Find the domain and the range of $f(x)$. **Fall 24-25**

$y = \frac{x+1}{x-2}$

$y(x-2) = x+1$

$yx - 2y = x+1$

$yx - x = 2y+1$

$x(y-1) = 2y+1$

$x = \frac{2y+1}{y-1}$

$f^{-1}(x) = \frac{2x+1}{x-1}$

$Df(x): \mathbb{R} / \{2\}$

$Rf(x) = Df^{-1}(x): \mathbb{R} / \{1\}$

Q4. Let $f(x) = 3x^7 + 2$. Find $f^{-1}(x)$ and its domain. **Fall 22-23**

$y = 3x^7 + 2$

$-3x^7 = -y + 2$

$x^7 = \frac{-y+2}{-3}$

$x = \sqrt[7]{\frac{-y+2}{-3}}$

$f^{-1}(x) = \sqrt[7]{\frac{-x+2}{-3}}$

$D: \mathbb{R}$

لأن الجذر فردي ($\sqrt[7]{\ }$)

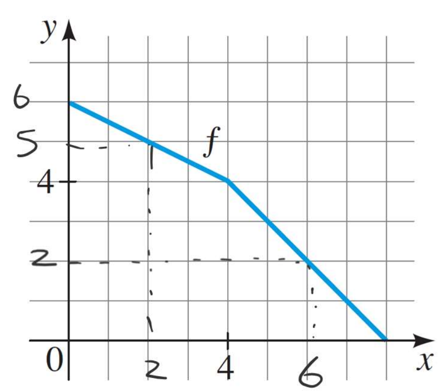
Q5. A graph of a function is given.

Use the graph to find the indicated values.

(a) $f^{-1}(2) = 6$

(b) $f^{-1}(5) = 2$

(c) $f^{-1}(6) = 0$



Q6. A table of values for a one-to-one function is given.

x	1	2	3	4	5	6
$f(x)$	4	6	2	5	0	1

Find:

(a) $f^{-1}(2)$ $= 3$

(b) $f^{-1}(0)$ $= 5$

Section 3.3

Spring 22-23

Q1. Use long divison to divide $P(x) = 27x^3 + 9x^2 - 3x - 9$ by $D(x) = 3x - 2$ and write the result in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$

$$
\begin{array}{r|l}
 & 9x^2 + 9x + 5 \\
\hline
3x-2 & 27x^3 + 9x^2 - 3x - 9 \\
 & -(27x^3 - 18x^2) \\
\hline
 & 27x^2 - 3x - 9 \\
 & -(27x^2 - 18x) \\
\hline
 & 15x - 9 \\
 & -(15x - 10) \\
\hline
 & 1
\end{array}
$$

$$\frac{P(x)}{D(x)} = 9x^2 + 9x + 5 + \frac{1}{3x-2}$$

Q2. For the two polynomials $P(x) = -x^3 - 2x + 6$ and $D(x) = x + 1$ are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$

$$
\begin{array}{r|rrrr}
-1 & -1 & 0 & -2 & 6 \\
 & & 1 & -1 & 3 \\
\hline
 & -1 & 1 & -3 & \boxed{9}
\end{array}
$$

$$\frac{P(x)}{D(x)} = -x^2 + x - 3 + \frac{9}{x+1}$$

Q3. Find the remainder of dividing $f(x) = x^{1001} + x^{50} + 100$ by $x - 1$ **Fall 24-25**

$f(1) = 1^{1001} + 1^{50} + 100$

$= 1 + 1 + 100 = 102$

$P(-2) = (-2)^3 - 5(-2)^2 - 8(-2) + 12 = 0$

Q4. Which of the following is a factor of $P(x) = x^3 - 5x^2 - 8x + 12$? **Fall 22-23**

(a) $x + 1$ (b) $x + 2$ (c) $x + 4$ (d) $x - 3$

Q5. Find a polynomial of degree 4 that has zeros $-3, 0, 2$, and 6, and the coefficient of x^4 is 2. **Fall 22-23**

$P(x) = a(x+3)\,x\,(x-2)(x-6)$

$= a(x^2+3x)(x^2 \underbrace{-6x-2x}_{-8x} + 12)$

$= a(x^4 - 8x^3 + 12x^2 + 3x^3 - 24x^2 + 36x)$

$= a(x^4 - 5x^3 - 12x^2 + 36x)$

coefficient of x^4 is 2

therefore $a = 2$

$P(x) = 2x^4 - 10x^3 - 24x^2 + 72x$

Section 3.4

Q1. Given $P(x) = x^3 - 6x^2 + 11x - 6$

Fall 24-25

a) Find all possible zeros.

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$$

b) Factor $P(x)$ completely.

$$P(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 0$$

1	1	-6	11	-6
		1	-5	6
	1	-5	6	0

$$\therefore P(x) = (x-1)(x^2 - 5x + 6)$$

$$= (x-1)(x-3)(x-2)$$

c) Find all zeros.

Zeros: $\{1, 2, 3\}$

Q2. Find all zeros of $P(x) = 2x^3 + x^2 - 7x - 6$ **Fall 22-23**

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 6$$

$$P(1) = 2(1)^3 + 1^2 - 7(1) - 6 = -8$$

$$P(2) = 2(2)^3 + 2^2 - 7(2) - 6 = 0$$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -7 & -6 \\ & & 4 & 10 & 6 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

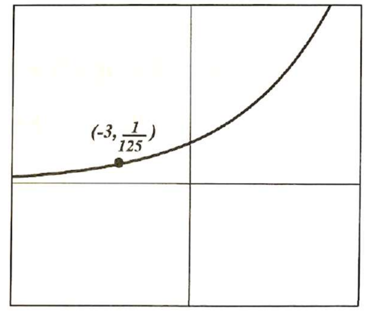
$$P(x) = (x-2)(2x^2 + 5x + 3)$$

$$= (x-2)(2x+3)(x+1)$$

zeros: $\left\{2, -\frac{3}{2}, -1\right\}$

Section 4.1, 4.2

Q1. Write the exponential function whose graph is given. **Fall 22-23**



$y = a^x$

$\frac{1}{125} = a^{-3}$

$\frac{1}{125} = \frac{1}{a^3}$

$\frac{1}{5^3} = \frac{1}{a^3} \Rightarrow \therefore a = 5 \Rightarrow f(x) = 5^x$

Q2. Find the range of $f(x) = 8^{5x+1}$ **Fall 22-23**

(a) $(8, \infty)$ (b) $\left(\frac{8}{5}, \infty\right)$ (c) $(0, \infty)$ (d) $\left(\frac{5}{8}, \infty\right)$

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Q3. Find the range of:

(a) $y = e^{x+1}$ $R: (0, \infty)$

(b) $y = e^x - 2$ $R: (-2, \infty)$