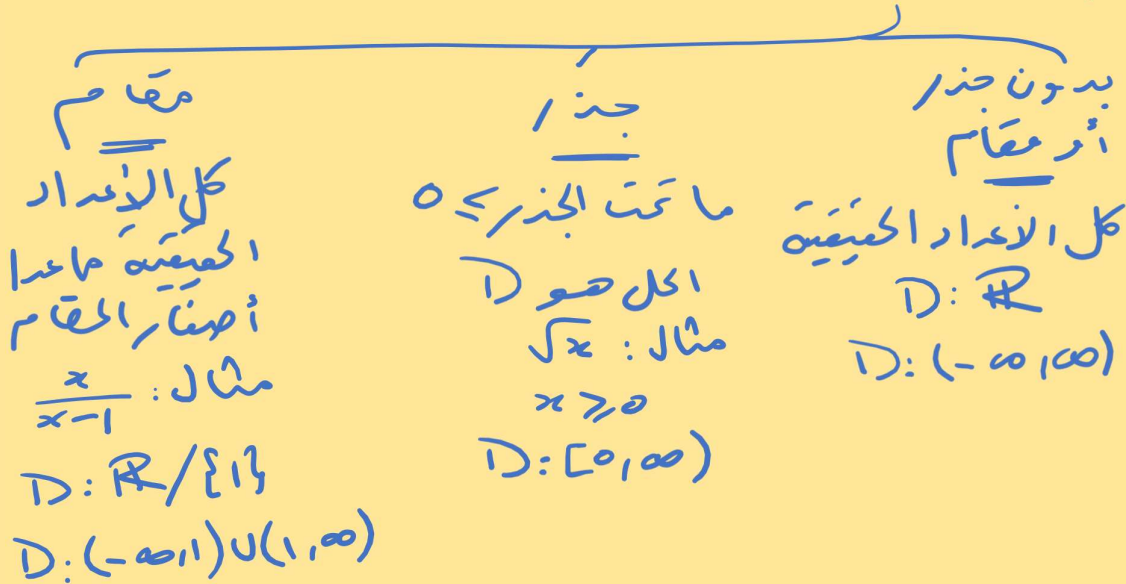


Section 2.1

الخلاصة

- لإيجاد مجال domain الدالة



- قيم evaluate الدالة معناها نأخذ ما بداخل الأقواس ونضعه مكان x في المعادلة

Problems

Q1. If $f(x) = 3x - 5$. Find $\frac{f(x)-f(a)}{x-a}$ and simplify

Fall 24-25 

$$f(a) = 3a - 5$$

$$\begin{aligned} \frac{f(x)-f(a)}{x-a} &= \frac{3x-5-(3a-5)}{x-a} \\ &= \frac{3x-\cancel{5}-3a+\cancel{5}}{x-a} \\ &= \frac{3x-3a}{x-a} \\ &= \frac{3(\cancel{x-a})}{\cancel{x-a}} = 3 \end{aligned}$$

Q2. Let $f(x) = x^2 + 2x$. Find $\frac{f(x)-f(2)}{x-2}$ and simplify

Fall 23-24 

$$f(2) = 2^2 + 2(2) = 4 + 4 = 8$$

$$\begin{aligned}\frac{f(x)-f(2)}{x-2} &= \frac{x^2 + 2x - 8}{x-2} \\ &= \frac{(x+4)(\cancel{x-2})}{\cancel{x-2}} = x+4\end{aligned}$$

Q3. Let $f(x) = x^2 - 5x$. Find $f(2a+1) - 2f(a)$.

Fall 22-23 

$$\begin{aligned}f(2a+1) &= (2a+1)^2 - 5(2a+1) \\ &= 4a^2 + 4a + 1 - 10a - 5 \\ &= 4a^2 - 6a - 4\end{aligned}$$

$$\begin{aligned}2f(a) &= 2(a^2 - 5a) \\ &= 2a^2 - 10a\end{aligned}$$

$$\begin{aligned}f(2a+1) - 2f(a) &= 4a^2 - 6a - 4 - (2a^2 - 10a) \\ &= 2a^2 + 4a - 4\end{aligned}$$

Q4. Let $f(x) = x^2 - 2x + 2$. Find the value of $\frac{f(3+h)-f(3)}{h}$, $h \neq 0$.

Spring 22-23 

$$\begin{aligned}f(3+h) &= (3+h)^2 - 2(3+h) + 2 \\ &= 9 + 6h + h^2 - 6 - 2h + 2 \\ &= h^2 + 4h + 5\end{aligned}$$

$$f(3) = 3^2 - 2(3) + 2 = 9 - 6 + 2 = 5$$

$$\frac{f(3+h)-f(3)}{h} = \frac{h^2 + 4h + \cancel{5} - \cancel{5}}{h} = \frac{\cancel{h}(h+4)}{\cancel{h}} = h+4$$

Q5. Let $f(x) = 3x^2 + 5x - 5$. Find the value of $\frac{f(2+h)-f(2)}{h}$, $h \neq 0$

Summer 22-23 

$$\begin{aligned} f(2+h) &= 3(2+h)^2 + 5(2+h) - 5 \\ &= 3(4+4h+h^2) + 10 + 5h - 5 \\ &= 12 + 12h + 3h^2 + 5 + 5h \\ &= 3h^2 + 17h + 17 \end{aligned}$$

$$f(2) = 3(2)^2 + 5(2) - 5 = 12 + 10 - 5 = 17$$

$$\begin{aligned} \frac{f(2+h)-f(2)}{h} &= \frac{3h^2 + 17h + \cancel{17} - \cancel{17}}{h} \\ &= \frac{\cancel{h}(3h+17)}{\cancel{h}} = 3h + 17 \end{aligned}$$

Q6. Consider the piecewise defined function

Spring 22-23 

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$$

Evaluate $5f(-1) + 2f(1)$

$$5f(-1) = 5\sqrt{-(-1)} = 5\sqrt{1} = 5$$

$$2f(1) = 2(1+1) = 2(2) = 4$$

$$5f(-1) + 2f(1) = 5 + 4 = 9$$

Q7. Consider the piecewise defined function

Summer 22-23 

$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } -2 \leq x \leq 2 \\ x^2 + x + 1 & \text{if } x > 3 \end{cases}$$

Evaluate $2f(0) + f(4) - 7$

$$2f(0) = 2\sqrt{9-0} = 2\sqrt{9} = 2(3) = 6$$

$$f(4) = 4^2 + 4 + 1 = 16 + 4 + 1 = 21$$

$$2f(0) + f(4) - 7 = 6 + 21 - 7 = 20$$

تدرب

(a) If $f(x) = x^2 + 2x$. Find $f(-3)$ and $f(-x)$ and simplify.

$$3, x^2 - 2x$$

(b) If $g(x) = \frac{1-x}{1+x}$. Find $g(a-1)$ and $g(x^2-1)$

$$\frac{2-a}{a}, \frac{2-x^2}{x^2}$$

Q8. Find the domain of $f(x) = \frac{\sqrt{36-x^2}}{x-6}$

Fall 22-23 

$$36 - x^2 \geq 0$$

$$-x^2 \geq -36$$

$$x^2 \leq 36$$

$$-6 \leq x \leq 6$$

$$[-6, 6)$$

$$x - 6 = 0$$

$$x = 6$$

$$\mathbb{R} / \{6\}$$

$$D: [-6, 6] \cap \mathbb{R} / \{6\}$$

$$D: [-6, 6)$$

Q9. Find the domain of $g(x) = \sqrt{x} + \sqrt{3-x}$

Fall 23-24 

$$x \geq 0$$

$$[0, \infty)$$

$$3-x \geq 0$$

$$-x \geq -3$$

$$x \leq 3$$

$$(-\infty, 3]$$

$$D: (-\infty, 3] \cap [0, \infty)$$

$$D: [0, 3]$$

Q10. Find the domain of $f(x) = \sqrt{x-1} - \frac{1}{\sqrt{2-x}}$

Spring 22-23 

$$x-1 \geq 0$$

$$x \geq 1$$

$$[1, \infty)$$

$$2-x > 0$$

$$-x > -2$$

$$x < 2$$

$$(-\infty, 2)$$

> بدون يساوي
لأنها في المقام

$$D: (-\infty, 2) \cap [1, \infty)$$

$$D: [1, 2)$$

Q11. Find the domain of $f(x) = \sqrt{5x-1} + \frac{4}{\sqrt{8-x}}$

Summer 22-23 

$$5x-1 \geq 0$$

$$5x \geq 1$$

$$x \geq \frac{1}{5}$$

$$[\frac{1}{5}, \infty)$$

$$8-x > 0$$

$$-x > -8$$

$$x < 8$$

$$(-\infty, 8)$$

$$D: (-\infty, 8) \cap [\frac{1}{5}, \infty)$$

$$D: [\frac{1}{5}, 8)$$

Q12. Find the domain $f(x) = 5x^2 + 4$, $0 \leq x \leq 2$

$$D: [0, 2]$$

تدرب

(a) Find the domain $f(x) = \frac{x+2}{\sqrt{x^2-1}}$

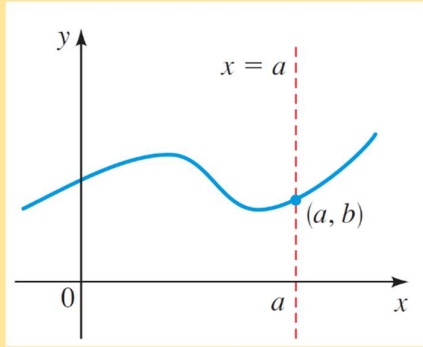
Section 2.2

الخلاصة

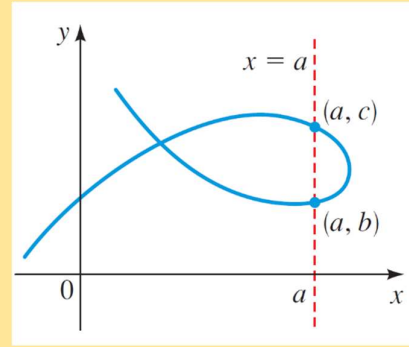
- لرسم دالة:

1. نختار 5 قيم عشوائية ل x ونحسب قيمة y لكل واحدة من القيم
2. نضع نقطة بناء على كل قيمة x و y
3. نوصل النقاط

- اختبار الدالة بالرسم: نرسم خط عمودي، إذا قطع رسم الدالة **أكثر من مرة** لا تكون دالة



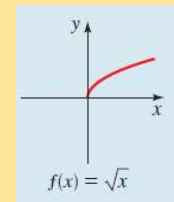
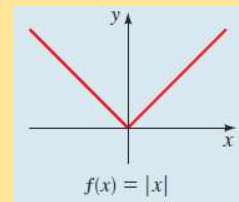
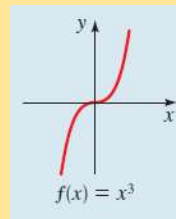
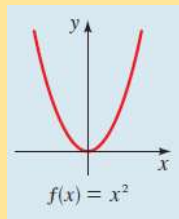
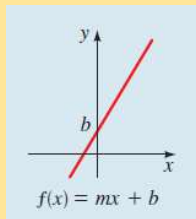
دالة



ليست دالة

- اختبار الدالة بالمعادلة: نتأكد أن أي قيمة ل x تقابلها قيمة واحدة فقط ل y

- رسومات دوال مهمة **حفظ**

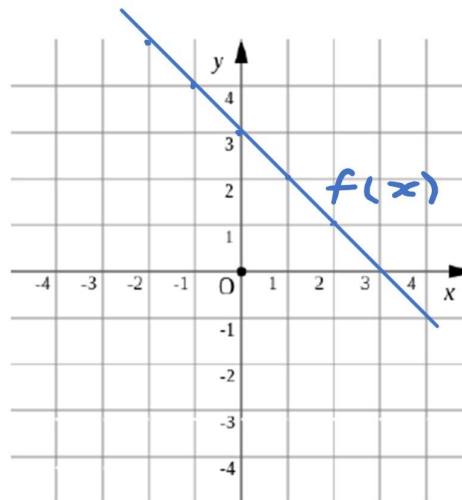


Problems

Q1. Sketch graphs of the following functions

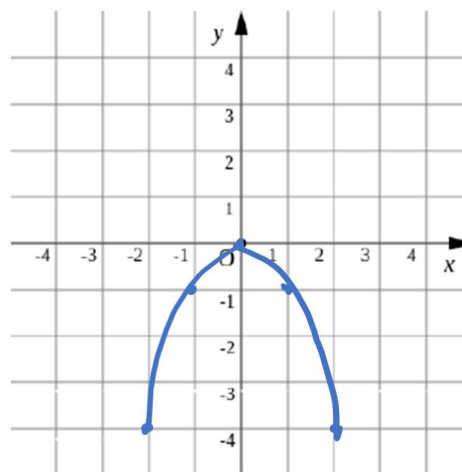
a) $f(x) = -x + 3$

x	$y = f(x)$
-2	$-(-2) + 3 = 5$
-1	$-(-1) + 3 = 4$
0	$-(0) + 3 = 3$
1	$-(1) + 3 = 2$
2	$-(2) + 3 = 1$



b) $h(x) = -x^2$

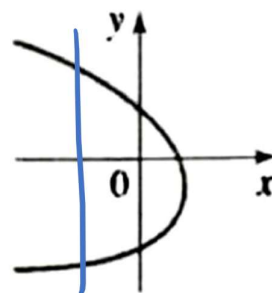
x	$y = f(x)$
-2	$-(-2)^2 = -4$
-1	$-(-1)^2 = -1$
0	0
1	$-(1)^2 = -1$
2	$-(2)^2 = -4$



Q2. Use the Vertical Line Test to determine whether the curve is a graph of a function of x .

Fall 23-24 

Not a function



Q3. Determine whether the equation defines y as a function of x

Summer 24-25 

$$x^2 - y - 6 = 1$$

$$-y - 6 = -x^2 + 1$$

$$-y = -x^2 + 1 + 6$$

$$-y = -x^2 + 7$$

$$y = x^2 - 7$$

function, any value of x gives one value of y

Q4. Determine whether the equation $\sqrt{x} - y^2 = 5$ defines y as a function of x .

Spring 22-23 

$$-y^2 = -\sqrt{x} + 5$$

$$y^2 = \sqrt{x} - 5$$

$$y = \pm \sqrt{\sqrt{x} - 5}$$

Not a function, any value of x gives two values of y ($\pm$)

Q5. Determine whether the equation $\frac{y}{2} + 8x^2 = 10$ defines y as a function of x .

Summer 22-23 

$$\frac{y}{2} = -8x^2 + 10$$

$$y = 2(-8x^2 + 10)$$

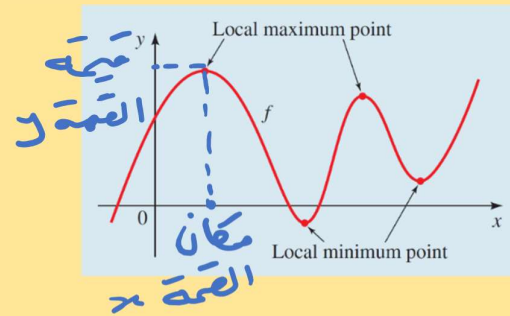
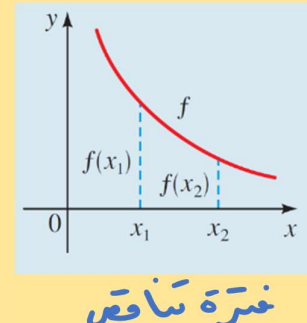
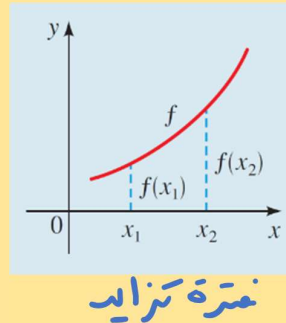
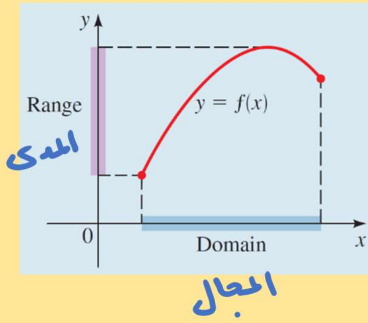
$$y = -16x^2 + 20$$

function

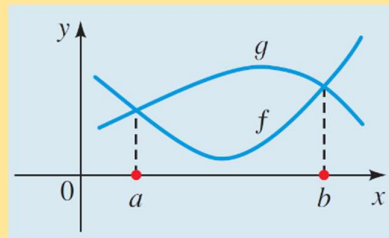
Section 2.3

الخلاصة

- نستطيع الحصول على كل المعلومات عن الدالة من الرسم الكامل لها

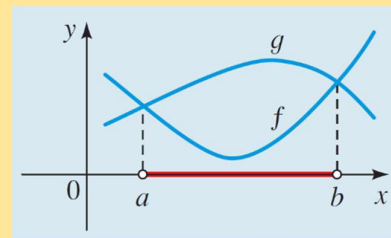


- نقدر نقارن بين قيم دالتين برسم الاثنين مع بعض



$$f(a) = g(a)$$

$$f(b) = g(b)$$



Problems

Q1. Given the graph of $f(x)$. Find the following

Summer 24-25 

a. $f(0) = 2$

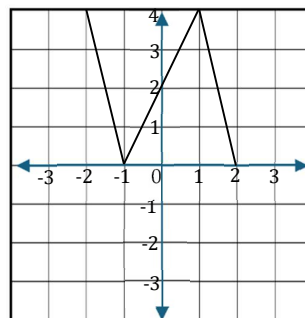
$f(1) = 4$

b. Domain of $f(x) = [-2, 2]$

c. Intervals on which $f(x)$ is increasing: $(-1, 1)$

d. Local maximum of $f(x) = 4$

e. Local minimum of $f(x) = 0$



Q2. Given the graph of function f , find:

Fall 24-25 

1. a) Local maximum 2

b) The value(s) of x at which the local maximum occurs 0

2. a) Local minimum -1

b) The value(s) of x at which the local minimum occurs $-1, 1$

3. a) Domain of $f(x) = (-\infty, \infty)$

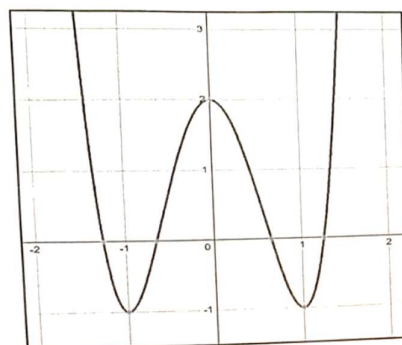
b) Range of $f(x) = [-1, \infty)$

4. a) Intervals for $f(x)$ increasing $(-1, 0) \cup (1, \infty)$

b) Intervals for $f(x)$ decreasing $(-\infty, -1) \cup (0, 1)$

5. a) Find $f^{-1}(2) = 0, -1.5, 1.5$

b) Find $f(-1) = -1$



Q3. The graph of a function f is given.

Fall 23-24 

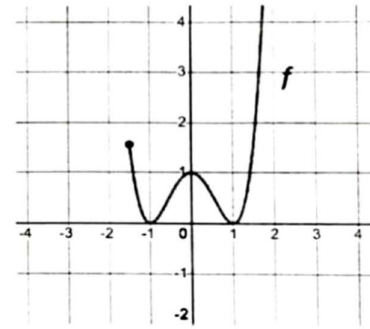
i. Find the intervals on which f is increasing.

$$(-1, 0) \cup (1, \infty)$$

ii. Find the local maximum value of f and the value of x at which it occurs.

$$\text{local maximum} = 1$$

$$\text{it occurs at } x = 0$$



Q4. Given the graphs of two functions f and g . Find:

Fall 22-23 

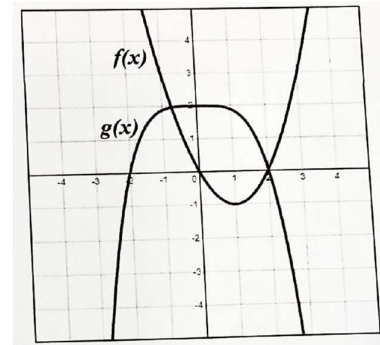
a. $f(-1) = 3$

b. Local minimum value of $f(x) = -1$

c. Values of x where $g(x) = 0$ $-2, 2$

d. Range of $g(x)$ $(-\infty, 2]$

e. Interval where $f(x)$ is increasing $(1, \infty)$



Q5. The graphs of f and g are given

Spring 22-23 

a) Find the interval on which f is decreasing $(-2, 2)$

b) Find the local minimum value of g 0

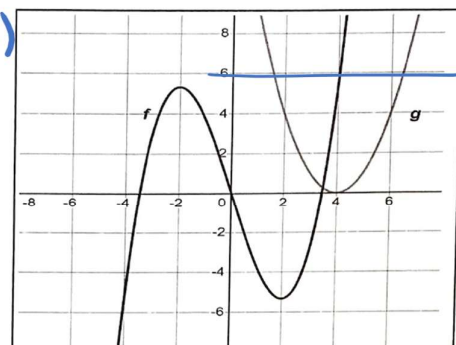
c) Find the value of x at which f has a local maximum -2

d) Determine whether f is even, odd or neither

Odd, symmetric about the origin

e) Is g one-to-one?

No, by Horizontal Line Test



Q6. Graph of the function f is given

Summer 22-23 

a) Find the interval on which f is decreasing

$(0, 2)$

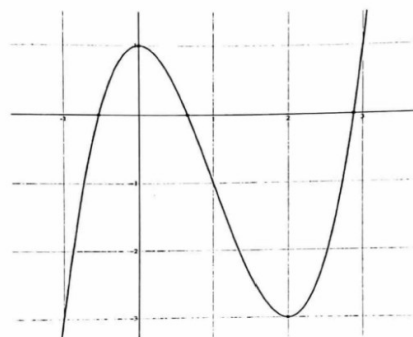
b) Find the local minimum value of f -3

c) Find the value of x at which f has local maximum

0

d) Determine whether f is even, odd, or neither

neither



Q7. Graphs of the functions f and g are given.

(a) Which is larger, $f(0)$ or $g(0)$?

$f(0)$

(b) Which is larger, $f(-3)$ or $g(-3)$?

$g(-3)$

(c) For which values of x is $f(x) = g(x)$?

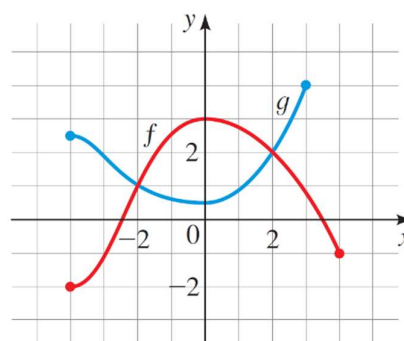
$-2, 2$

(d) Find the values of x for which $f(x) \leq g(x)$.

$[-4, -2] \cup [2, 3]$

(e) Find the values of x for which $f(x) > g(x)$.

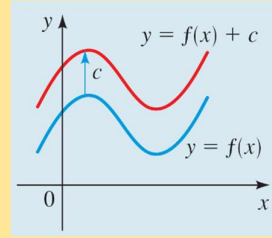
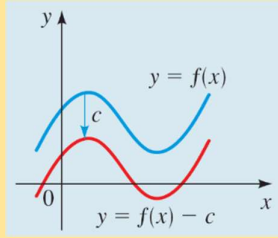
$(-2, 2)$



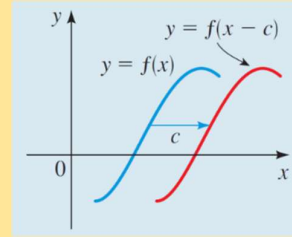
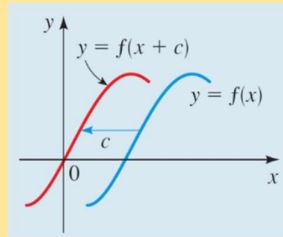
Section 2.6

الخلاصة

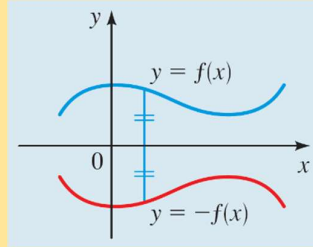
- عند إضافة ثابت (رقم) للدالة يتسبب في تحرك رسم الدالة عموديا



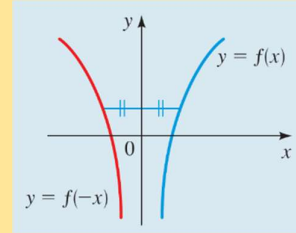
- عند إضافة رقم إلى x داخل الدالة يتحرك رسم الدالة أفقيا



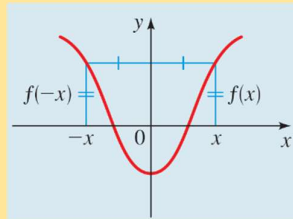
إذا عكسنا إشارة الدالة
تنعكس الرسمة حول x



إذا عكسنا إشارة x
تنعكس الرسمة حول y

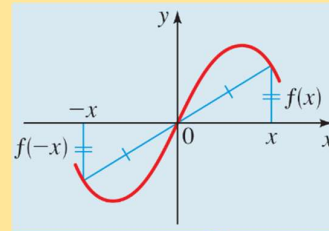


الدالة الزوجية: دالة لا تتغير بتغير
إشارة متغير الدالة $f(-x) = f(x)$



متماثلة حول y

الدالة الفردية: تنعكس إشارة الدالة بالكامل
بتغير إشارة متغير الدالة $f(-x) = -f(x)$



متماثلة حول
origin

Problems

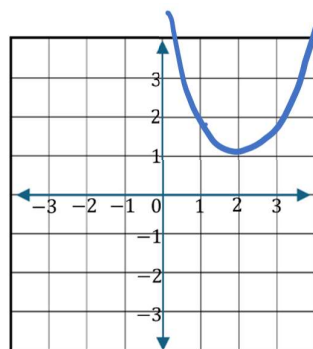
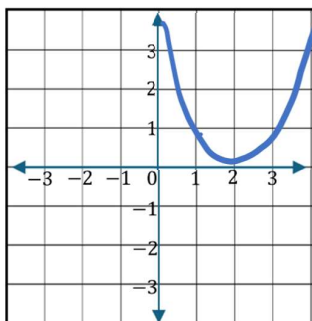
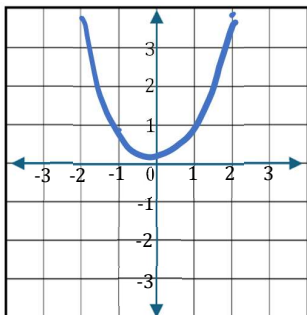
Q1. Sketch each of the following Functions

Summer 24-25 

a. $f(x) = x^2$

b. $f(x) = (x - 2)^2$

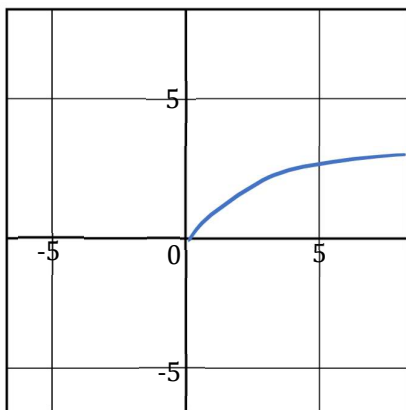
c. $f(x) = (x - 2)^2 + 1$



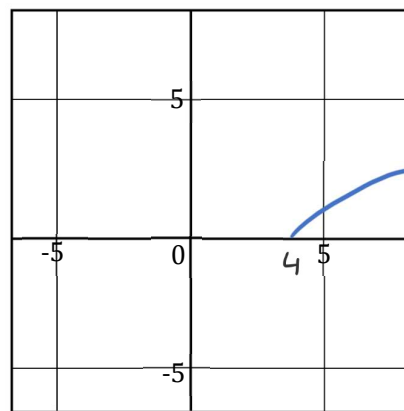
Q2. Sketch the graph of each of the following functions:

Fall 24-25 

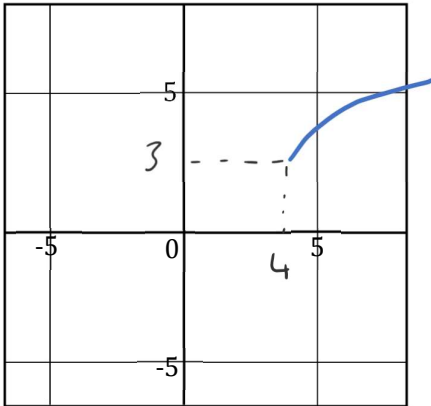
a. $f(x) = \sqrt{x}$



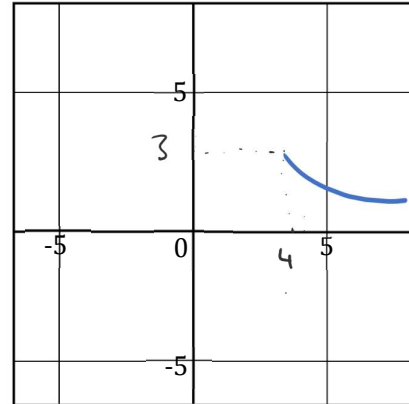
b. $f(x) = \sqrt{x - 4}$



c. $f(x) = \sqrt{x-4} + 3$



d. $f(x) = -\sqrt{x-4} + 3$



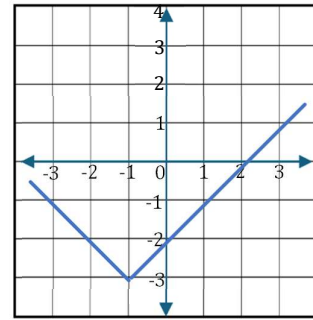
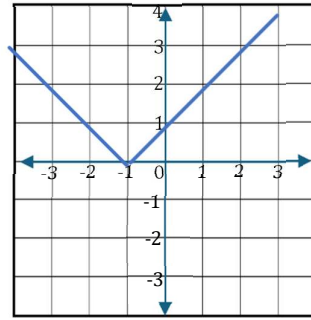
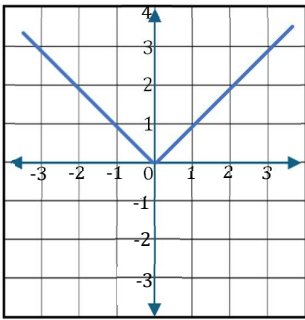
Q3. Sketch the graph of each of the following functions

Fall 23-24 

i. $f(x) = |x|$

ii. $f(x) = |x + 1|$

iii. $f(x) = |x + 1| - 3$



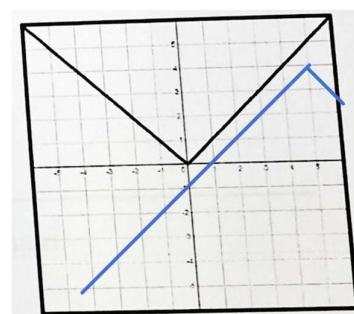
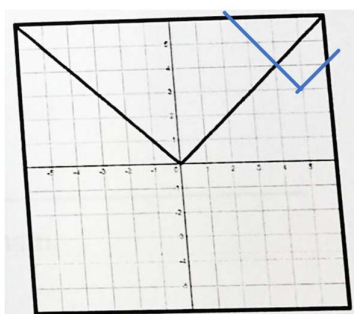
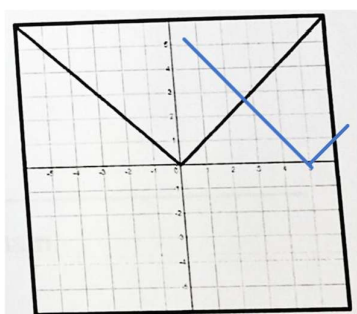
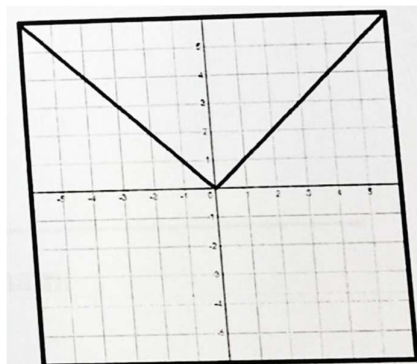
Q4. The graph of $f(x) = |x|$ is given on the right. Sketch the graph of each of the following functions:

Fall 22-23 

a. $f(x) = |x - 4|$

b. $f(x) = |x - 4| + 3$

c. $f(x) = -|x - 4| + 3$



Q5. The graph of $y = f(x)$ is given. Match each equation with its graph.

(a) $y = f(x - 4)$

③

(b) $y = f(x) + 3$

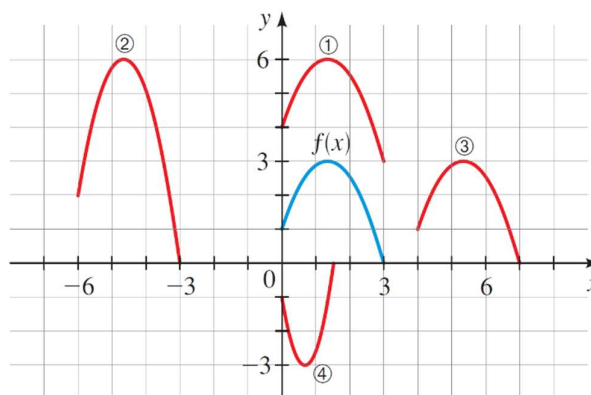
①

(c) $y = 2f(x + 6)$

②

(d) $y = -f(2x)$

④



Q6. Determine whether the function is even, odd, or neither
 $f(x) = x^3 + 2x$

Summer 24-25 

$$\begin{aligned} f(-x) &= (-x)^3 + 2(-x) \\ &= -x^3 - 2x \\ &= -(x^3 + 2x) \\ &= -f(x) \end{aligned}$$

The function is odd

Q7. Determine whether the function $f(x) = x^3 - x$ is even, odd, or neither. Fall 23-24 

$$\begin{aligned} f(-x) &= (-x)^3 - (-x) \\ &= -x^3 + x \\ &= -(x^3 - x) \\ &= -f(x) \end{aligned}$$

The function is odd

Section 2.7

الخلاصة

- يمكن جمع، طرح، ضرب، و قسمة الدوال. و ينتج عنها دالة جديدة مداها domain هو تقاطع intersection مدى الدالتين $A \cap B$

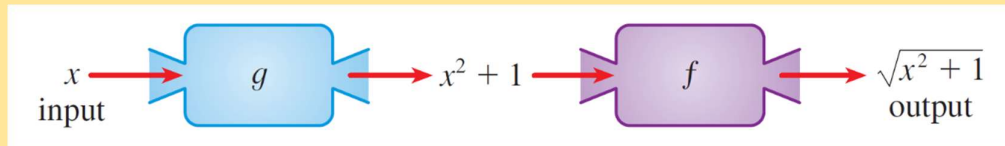
- الدالة المركبة تتكون من دالتين بحيث تستخدم مخرجات واحدة كمدخلات للدالة الأخرى

مثال

$$f(x) = \sqrt{x}, \quad g(x) = x^2 + 1$$

$$f \circ g = f(g(x)) = \sqrt{g(x)}$$

$$= \sqrt{x^2 + 1}$$



Problems

Q1. Let $f(x) = x - 2$, $g(x) = x^2 - 4$. Evaluate the following:

Summer 24-25

a. $\left(\frac{f}{g}\right)(3)$

$$\frac{f}{g} = \frac{x-2}{x^2-4}$$

$$\left(\frac{f}{g}\right)(3) = \frac{3-2}{3^2-4} = \frac{1}{9-4} = \frac{1}{5}$$

b. $(f \cdot g)(1)$

$$f \cdot g = (x-2)(x^2-4)$$

$$f \cdot g(1) = (1-2)(1^2-4) = (-1)(-3) = 3$$

Q2. Let $f(x) = 1 + \frac{1}{x}$ and $g(x) = x + 1$. Find $\left(\frac{f}{g}\right)(1)$

Spring 22-23 

$$\frac{f}{g} = \frac{1 + \frac{1}{x}}{x+1}$$

$$\left(\frac{f}{g}\right)(1) = \frac{1 + \frac{1}{1}}{1+1} = \frac{2}{2} = 1$$

Q3. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+4}$, find

Fall 23-24 

i. $(f \circ g)(x)$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= (\sqrt{x+4})^2 - 1 = x+4 - 1 = x+3 \end{aligned}$$

ii. $g(f(1))$

$$\begin{aligned} f(1) &= 1^2 - 1 = 0 \\ g(f(1)) &= g(0) = \sqrt{0+4} = \sqrt{4} = 2 \end{aligned}$$

Q4. Let $f(x) = \frac{4}{x-3}$ and $g(x) = 2x^2 + 1$. Find $(f \circ g)(2)$

Fall 22-23 

(a) $\frac{1}{3}$

(b) -3

(c) $\frac{2}{3}$

(d) 12

$$\begin{aligned} g(2) &= 2(2)^2 + 1 \\ &= 9 \\ f(9) &= \frac{4}{9-3} = \frac{4}{6} \end{aligned}$$

Q5. Let $f(x) = 4 + \frac{8}{x}$ and $g(x) = 2x + 4$. Find $f(g(-1))$

Summer 22-23 

$$g(-1) = 2(-1) + 4 = -2 + 4 = 2$$

$$f(2) = 4 + \frac{8}{2} = 4 + 4 = 8$$

Q6. Evaluate the following using the table.

Summer 24-25 

x	-1	1	2	3	4
$f(x)$	0	2	3	4	5
$g(x)$	2	2	5	10	17

a. $f^{-1}(0) = -1$

b. $(g \circ f)(3) = g(4) = 17$

c. $(f \circ g)(1) = f(2) = 3$

d. $2f(2) - g(-1) = 2(3) - 2 = 6 - 2 = 4$

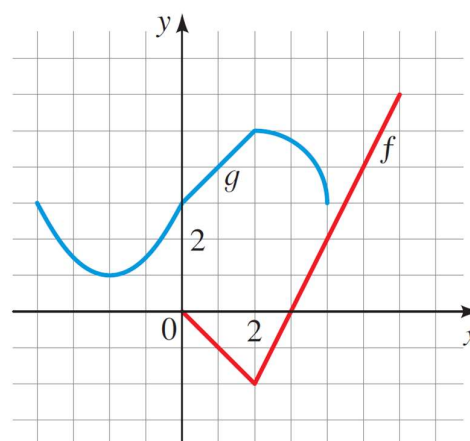
e. $f(f^{-1}(4)) = 4$

Q7. Use the given graphs of f and g to evaluate the expression.

(a) $f(g(2)) = f(5) = 4$

(b) $(g \circ f)(4) = g(2) = 5$

(c) $(g \circ g)(-2) = g(1) = 4$

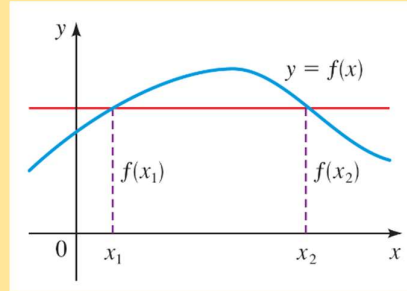


Section 2.8

الخلاصة

- لمعرفة إذا كانت الدالة one-to-one من الرسم، نرسم خط أفقي: إذا قطع منحنى الدالة أكثر من مرة لا تكون

one-to-one



- لإيجاد دالة عكسية inverse function

1- اكتب y بدلا من $f(x)$

2- أوجد x كمعادلة في y (نعيد ترتيب المعادلة بحيث تكون x فقط على اليسار)

3- اكتب x مكان y و $f^{-1}(x)$ مكان x

Problems

Q1. Determine whether the function $f(x) = -2x + 4$ is one-to-one

Fall 23-24

$$f(a) = f(b)$$

$$-2a + 4 = -2b + 4$$

$$-4$$

$$-2a = -2b$$

$$\div -2$$

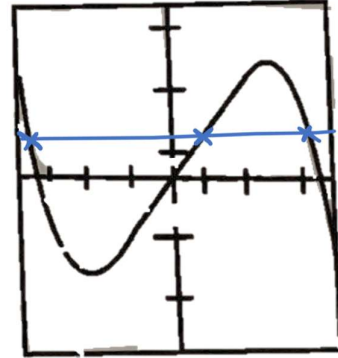
$$a = b$$

The function is one-to-one

Q3. Determine whether the function is one to one or not

Summer 24-25 

Not one-to-one
by Horizontal Line Test

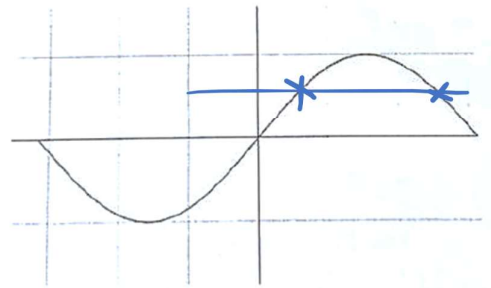


Q4. From the following graph

Fall 24-25 

a) Determine whether the graph represents the graph of a function

By Vertical Line Test
the graph represents
a function



b) Determine whether $f(x)$ is one-to-one or not

By Horizontal Line Test, the function is
Not one-to-one

Q5. For $f(x) = \frac{3x-2}{x-1}$, find:

Summer 24-25 

a) Domain of $f(x)$

$$\begin{aligned} x-1 &= 0 \\ x &= 1 \\ D: \mathbb{R} / \{1\} \end{aligned}$$

حل آخر

$$D: (-\infty, 1) \cup (1, \infty)$$

b) $f^{-1}(x)$

$$\begin{aligned} y &= \frac{3x-2}{x-1} \\ y(x-1) &= 3x-2 \\ yx-y &= 3x-2 \\ yx-3x &= y-2 \\ x(y-3) &= y-2 \\ x &= \frac{y-2}{y-3} \\ f^{-1}(x) &= \frac{x-2}{x-3} \end{aligned}$$

Q6. Find the inverse of the function $f(x) = \frac{x+1}{x-2}$. Find the domain and the range of $f(x)$.

Fall 24-25 

$$\begin{aligned} y &= \frac{x+1}{x-2} \\ y(x-2) &= x+1 \\ yx-2y &= x+1 \\ yx-x &= 2y+1 \\ x(y-1) &= 2y+1 \end{aligned}$$

$$x = \frac{2y+1}{y-1}$$

$$f^{-1}(x) = \frac{2x+1}{x-1}$$

$$Df(x): \mathbb{R} / \{2\}$$

$$Rf(x) = Df^{-1}(x): \mathbb{R} / \{1\}$$

Q7. Let $f(x) = \frac{-3x-1}{x+5}$

Fall 23-24 

i. Find the inverse function f .

$$y = \frac{-3x-1}{x+5}$$

$$y(x+5) = -3x-1$$

$$yx + 5y = -3x - 1$$

$$yx + 3x = -5y - 1$$

$$x(y+3) = -5y-1$$

$$x = \frac{-5y-1}{y+3}$$

$$f^{-1}(x) = \frac{-5x-1}{x+3}$$

ii. Find the range of f .

$$R_f = D_{f^{-1}} : \mathbb{R} / \{-3\}$$

Q8. Let $f(x) = 3x^7 + 2$. Find $f^{-1}(x)$ and its domain.

Fall 22-23 

$$y = 3x^7 + 2$$

$$-3x^7 = -y + 2$$

$$x^7 = \frac{-y+2}{-3}$$

$$x = \sqrt[7]{\frac{-y+2}{-3}}$$

$$f^{-1}(x) = \sqrt[7]{\frac{-x+2}{-3}}$$

$$D: \mathbb{R}$$

لأن الجذر خدي (7)

Q9. Let $f(x) = \frac{3x-5}{2-x}$

Spring 22-23 

a) Find the inverse function of f

$$y = \frac{3x-5}{2-x}$$

$$y(2-x) = 3x-5$$

$$2y - xy = 3x - 5$$

$$-xy - 3x = -2y - 5$$

$$x(-y-3) = -2y-5$$

$$x = \frac{-2y-5}{-y-3} = \frac{2y+5}{y+3}$$

$$f^{-1}(x) = \frac{2x+5}{x+3}$$

b) Find the range of f

$$R_f = \text{Dom } f^{-1}: \mathbb{R} / \{-3\}$$

Q10. Let $f(x) = \frac{8x+1}{4+x}$

Summer 22-23 

a) Find the inverse function of $f(x)$

$$y = \frac{8x+1}{4+x}$$

$$y(4+x) = 8x+1$$

$$4y + xy = 8x+1$$

$$xy - 8x = -4y+1$$

$$x(y-8) = -4y+1$$

$$x = \frac{-4y+1}{y-8}$$

$$f^{-1}(x) = \frac{-4x+1}{x-8}$$

b) Find the range of $f(x)$

$$R_f = \text{Dom } f^{-1}: \mathbb{R} / \{8\}$$

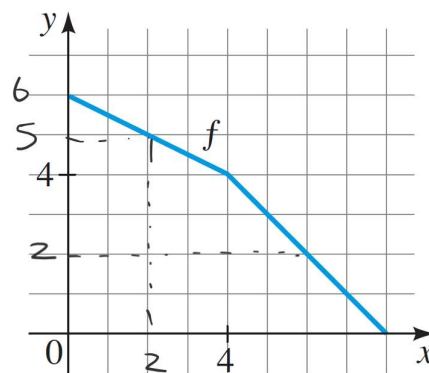
Q11. A graph of a function is given.

Use the graph to find the indicated values.

(a) $f^{-1}(2) = 6$

(b) $f^{-1}(5) = 2$

(c) $f^{-1}(6) = 0$



Q12. A table of values for a one-to-one function is given.

x	1	2	3	4	5	6
$f(x)$	4	6	2	5	0	1

Find:

(a) $f^{-1}(2) = 3$

(b) $f^{-1}(0) = 5$

Section 3.3

الخلاصة

- يمكن قسمة دالة كثيرة حدود على دالة كثيرة حدود بنفس طريق قسمة رقم على رقم، و تسمى هذه الطريقة:

القسمة المطولة long division

- القسمة التركيبية Synthetic division

نقدر نستخدمها فقط إذا كان المقسوم عليه على صورة $x - c$ (جمع أو طرح)

- نقدر نكتب القسمة كالتالي:

$$\frac{\text{Dividend} \text{ (المقسوم)}}{\text{Divisor} \text{ (المقسوم عليه)}} = \text{Quotient} \text{ (ناتج القسمة)} + \frac{\text{Remainder} \text{ (المبقي)}}{D(x)}$$

- Remainder Theorem:

- إذا كان المقسوم عليه divisor على صورة $x - c$ فإن متبقي القسمة يساوي $P(c)$

- Factor Theorem:

- إذا كانت $P(c) = 0$ فإن c هي أحد أصفار الدالة $P(x)$ وبالتالي فإن $x - c$ هي أحد معاملات (factors) $P(x)$

يعني $P(x)$ تقبل القسمة على $(x - c)$ بدون مَبَقِي

Problems

Q1. Use long division to divide $P(x) = 27x^3 + 9x^2 - 3x - 9$ by

Spring 22-23 

$D(x) = 3x - 2$ and write the result in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$

$$\begin{array}{r}
 9x^2 + 9x + 5 \\
 3x - 2 \overline{) 27x^3 + 9x^2 - 3x - 9} \\
 \underline{- 27x^3 + 18x^2} \\
 27x^2 - 3x - 9 \\
 \underline{- 27x^2 + 18x} \\
 15x - 9 \\
 \underline{- 15x + 10} \\
 1
 \end{array}$$

$$\frac{P(x)}{D(x)} = 9x^2 + 9x + 5 + \frac{1}{3x - 2}$$

Q2. Use long division to divide $P(x) = 36x^3 - 18x^2 + 8x - 2$ by
 $D(x) = 6x - 2$

Summer 22-23 

$$\begin{array}{r}
 6x^2 - x + 1 \\
 6x - 2 \overline{) 36x^3 - 18x^2 + 8x - 2} \\
 \underline{- 36x^3 + 12x^2} \\
 -6x^2 + 8x - 2 \\
 \underline{+ 6x^2 - 2x} \\
 6x - 2 \\
 \underline{6x - 2} \\
 0
 \end{array}$$

$$\frac{36x^3 - 18x^2 + 8x - 2}{6x - 2} = 6x^2 - x + 1$$

Q3. For the two polynomials $P(x) = -x^3 - 2x + 6$ and $D(x) = x + 1$ are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$

$$\begin{array}{r|rrrrr} -1 & -1 & 0 & -2 & 6 & \\ & & 1 & -1 & 3 & \\ \hline & -1 & 1 & -3 & 9 & \end{array}$$

$$\frac{P(x)}{D(x)} = -x^2 + x - 3 + \frac{9}{x+1}$$

Q4. Find the quotient and remainder using long division

$$\frac{x^3 + 2x + 1}{x^2 - x + 3}$$

$$\begin{array}{r} x+1 \\ x^2-x+3 \overline{) x^3 + 2x + 1} \\ \underline{-x^3 + x^2 + 3x} \\ x^2 - x + 1 \\ \underline{-x^2 + x + 3} \\ -2 \end{array}$$

$$\frac{x^3 + 2x + 1}{x^2 - x + 3} = x + 1 - \frac{2}{x^2 - x + 3}$$

تدرب

Use long division to divide $P(x) = 8x^4 + 4x^3 + 6x^2$ by $D(x) = 2x^2 + 1$

Q5. Find the remainder of dividing $f(x) = x^{1001} + x^{50} + 100$ by $x - 1$

Fall 24-25 

$$\begin{aligned} f(1) &= 1^{1001} + 1^{50} + 100 \\ &= 1 + 1 + 100 = 102 \end{aligned}$$

Q6. What is the remainder when the polynomial $P(x) = 5x^{27} + 3x^3 - 5x - 1$ is divided by $x + 1$?

Fall 23-24 

$$\begin{aligned} P(-1) &= 5(-1)^{27} + 3(-1)^3 - 5(-1) - 1 \\ &= -5 - 3 + 5 - 1 = -4 \end{aligned}$$

Q7. What is the remainder of dividing $P(x) = 2x^3 - 5x^2 - 6x + 16$ by $x - 3$?

$$2(3)^3 - 5(3)^2 - 6(3) + 16$$

Fall 22-23 

(a) 7

(b) -65

(c) 43

(d) 3

Q8. Which of the following is a factor of $P(x) = x^3 - 5x^2 - 8x + 12$?

$$P(-2) = (-2)^3 - 5(-2)^2 - 8(-2) + 12 = 0$$

Fall 22-23 

(a) $x + 1$

(b) $x + 2$

(c) $x + 4$

(d) $x - 3$

Q9. Use the Factor Theorem to determine whether $(x + 1)$ is a factor of $P(x) = 2x^{101} - 3x^8 - 13x - 8$. Spring 22-23 

$$\begin{aligned} P(-1) &= 2(-1)^{101} - 3(-1)^8 - 13(-1) - 8 \\ &= -2 - 3 + 13 - 8 = 0 \end{aligned}$$

$x + 1$ is a factor of $P(x)$

Q10. Use the Factor Theorem to determine whether $(x + 1)$ is a factor of $P(x) = 3x^7 + 5x^6 + 6x + 4$

yes

نعم

Summer 22-23 

Q11. Find a polynomial of degree 4 that has zeros $-3, 0, 2$, and 6 , and the coefficient of x^4 is 2 .

Fall 22-23 

$$\begin{aligned}
 P(x) &= a(x+3)(x-2)(x-6) \\
 &= a(x^2+3x)(x^2-8x+12) \\
 &= a(x^4-8x^3+12x^2+3x^3-24x^2+36x) \\
 &= a(x^4-5x^3-12x^2+36x)
 \end{aligned}$$

coefficient of x^4 is 2
therefore $a = 2$

$$P(x) = 2x^4 - 10x^3 - 24x^2 + 72x$$

Section 3.4

الخلاصة

- الحلول (الأصفار) المحتملة لأي دالة كثيرة الحدود يجب أن تكون ناتجة عن قسمة تحليل الحد الثابت (factors of the constant term) على تحليل معامل أول حد (factors of leading coefficient)

leading coefficient

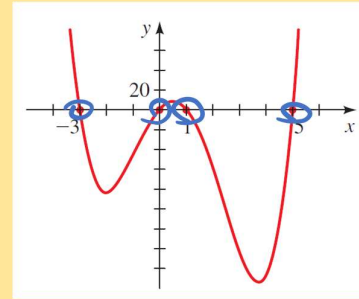
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$$\text{Possible rational zeros} = \frac{\text{Factors of the constant term}}{\text{Factors of the leading coefficient}}$$

- نستطيع إيجاد حلول (أصفار) الدالة باتباع الخطوات التالية:

- 1- إيجاد الأصفار المحتملة
- 2- نستخدم القسمة التركيبية لاختبار الأصفار المحتملة من الخطوة السابقة
- 3- نعيد الخطوة السابقة حتى نصل إلى معادلة تربيعية ثم نقوم بتحليلها بالطرق التقليدية

أصفار الدالة من الرسم
هي نقاط التقاطع مع x
مثال: -3, 0, 1, 5



ProblemsQ1. Given $P(x) = x^3 - 6x^2 + 11x - 6$ Fall 24-25 

a) Find all possible zeros.

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$$

b) Factor $P(x)$ completely.

$$P(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 0$$

$$\begin{array}{r} 1 \quad 1 \quad -6 \quad 11 \quad -6 \\ \quad \quad 1 \quad -5 \quad 6 \\ \hline 1 \quad -5 \quad 6 \quad 0 \end{array}$$

$$\begin{aligned} \therefore P(x) &= (x-1)(x^2-5x+6) \\ &= (x-1)(x-3)(x-2) \end{aligned}$$

c) Find all zeros.

$$\text{Zeros: } \{1, 2, 3\}$$

Q2. Find all zeros of $P(x) = x^3 + 6x^2 + 5x - 12$

Fall 23-24 

$$\frac{P}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$P(1) = 1^3 + 6(1)^2 + 5(1) - 12 = 0$$

$$\begin{array}{r} 1 \overline{) 1 \quad 6 \quad 5 \quad -12} \\ \underline{ 1 \quad 7 \quad 12} \\ 1 \quad 7 \quad 12 \quad 0 \end{array}$$

$$\begin{aligned} P(x) &= (x-1)(x^2 + 7x + 12) \\ &= (x-1)(x+3)(x+4) \end{aligned}$$

Q3. Find all zeros of $P(x) = 2x^3 + x^2 - 7x - 6$

Fall 22-23 

$$\frac{P}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$$

$$P(1) = 2(1)^3 + 1^2 - 7(1) - 6 = -8$$

$$P(2) = 2(2)^3 + 2^2 - 7(2) - 6 = 0$$

$$\begin{array}{r} 2 \overline{) 2 \quad 1 \quad -7 \quad -6} \\ \underline{ 4 \quad 10 \quad 6} \\ 2 \quad 5 \quad 3 \quad 0 \end{array}$$

$$\begin{aligned} P(x) &= (x-2)(2x^2 + 5x + 3) \\ &= (x-2)(2x+3)(x+1) \end{aligned}$$

$$\text{zeros: } \left\{ 2, -\frac{3}{2}, -1 \right\}$$

Q4. Given $P(x) = x^3 - 3x^2 - x + 3$. Find all zeros of $P(x)$.

Summer 22-23 

$$\frac{P}{q} = \pm 1, \pm 3$$

نتیجه

Q5. Given that $x = 3$ is a zero of $P(x) = x^3 - 9x^2 + 26x - 24$. Find the other zeros of $P(x)$.

Spring 22-23 

$$\begin{array}{r} 3 \overline{) 1 \quad -9 \quad 26 \quad -24} \\ \underline{ 3 \quad -18 \quad 24} \\ 1 \quad -6 \quad 8 \quad 0 \end{array}$$

$$\begin{aligned} P(x) &= (x-3)(x^2 - 6x + 8) \\ &= (x-3)(\quad)(\quad) \end{aligned}$$

$$\text{zeros: } \{ \quad \}$$

نتیجه

Section 4.1, 4.2

الخلاصة

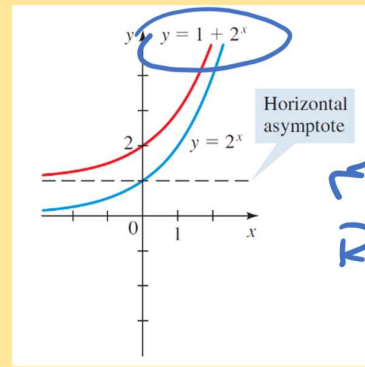
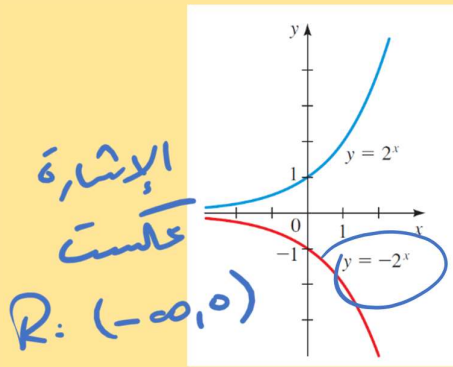
- The exponential function الدالة الأسية:

$$f(x) = a^x$$

where $a > 0$ and $a \neq 1$

لاحظ أن في الدالة الأسية يكون الأساس رقم ثابت و المتغير x يكون في الأس exponent

- مدى الدالة الأسية: $(0, -\infty)$



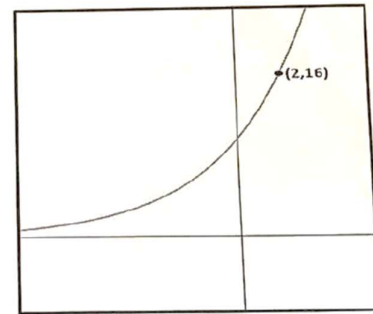
يغير الـ range فقط إذا تغير الأساس
ولا يغير بتغير الأس

Problems

Q1. Write the logarithmic function whose graph is given

Fall 24-25 

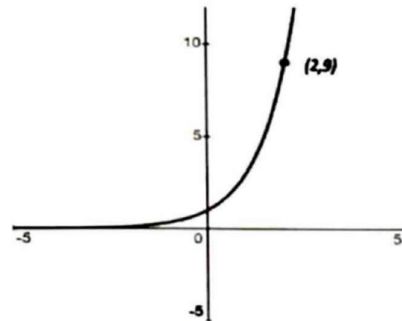
$$\begin{aligned}
 y &= a^x \\
 16 &= a^2 \\
 4^2 &= a^2 \\
 \therefore a &= 4 \\
 f(x) &= 4^x
 \end{aligned}$$



Q2. Find the exponential function whose graph is given.

Fall 23-24 

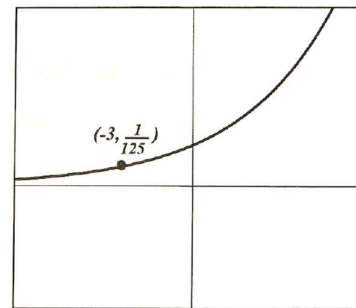
$$\begin{aligned}
 y &= a^x \\
 9 &= a^2 \\
 3^2 &= a^2 \\
 \therefore a &= 3 \\
 f(x) &= 3^x
 \end{aligned}$$



Q3. Write the exponential function whose graph is given.

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$$\begin{aligned}
 y &= a^x \\
 \frac{1}{125} &= a^{-3} \\
 \frac{1}{125} &= \frac{1}{a^3} \\
 \frac{1}{5^3} &= \frac{1}{a^3} \Rightarrow \therefore a = 5 \Rightarrow f(x) = 5^x
 \end{aligned}$$



Q4. Find the range of $f(x) = 8^{5x+1}$

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(a) $(8, \infty)$

(b) $\left(\frac{8}{5}, \infty\right)$

(c) $(0, \infty)$

(d) $\left(\frac{5}{8}, \infty\right)$

تذكر: المدى لا يتغير إلا بتغير الأس

Q5. Find the range of:

(a) $y = e^{x+1}$

$R: (0, \infty)$

(b) $y = e^x - 2$

$R: (-2, \infty)$

تدرب

Find the range of:

(a) $y = e^x$

(b) $y = e^x + 2$

(c) $y = 3^{-x}$

(d) $y = -3^x$