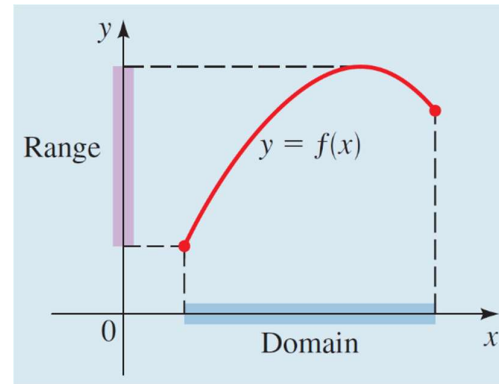


Section 2.3 – Getting Information from the graph of a Function

- A complete graph of a function contains all the information about the function

نستطيع الحصول على كل المعلومات عن الدالة من الرسم الكامل لها

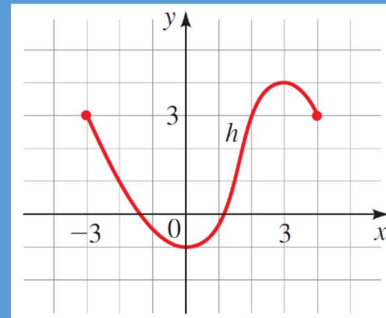
مثال: استطعنا تحديد المدى Domain و المجال Range من قيم x و y في الرسم.



Example 1

The graph of a function h is given.

- Find $h(-2)$, $h(0)$, $h(2)$, and $h(3)$.
- Find the domain and range of h .
- Find the values of x for which $h(x) = 3$.
- Find the values of x for which $h(x) \leq 3$.



Solution

$$(a) h(-2) = 1$$

$$h(0) = -1$$

$$h(2) = 3$$

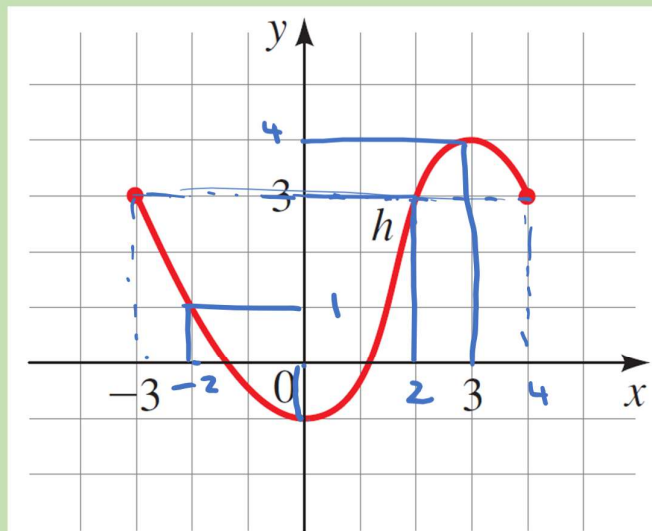
$$h(3) = 4$$

$$(b) D: [-3, 4]$$

$$R: [-1, 4]$$

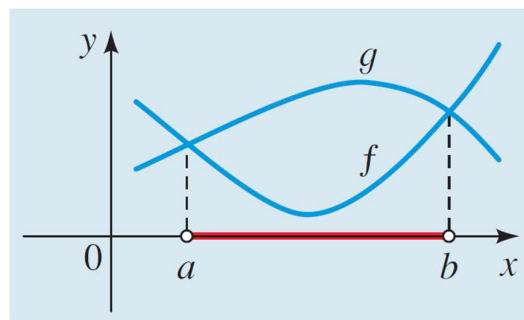
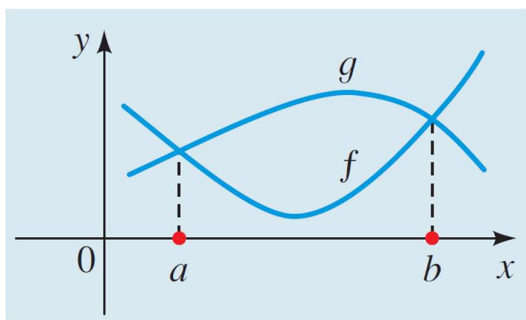
$$(c) h(-3) = h(2) = h(4) = 3$$

$$(d) h([-3, 2] \cup \{4\}) \leq 3$$



- We can compare the values of two functions by plotting both on the same graph

نقدر نقارن بين قيم دالتين برسم الاثنين مع بعض



الدالتان متساويتان عند نقاط التقاطع، بمعنى:

$$f(a) = g(a)$$

في الفترة (a, b) الدالة $g(x)$ أكبر من $f(x)$

$$g(x) > f(x)$$

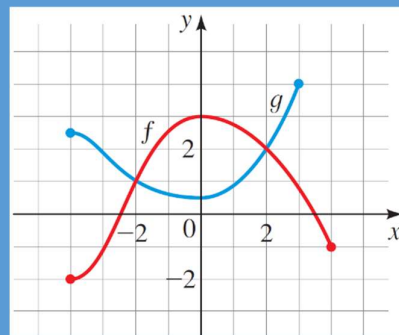
9

$$f(b) = g(b)$$

Example 2

Graphs of the functions f and g are given.

- Which is larger, $f(0)$ or $g(0)$?
- Which is larger, $f(-3)$ or $g(-3)$?
- For which values of x is $f(x) = g(x)$?
- Find the values of x for which $f(x) \leq g(x)$.
- Find the values of x for which $f(x) > g(x)$.



Solution

$$(a) f(0) > g(0)$$

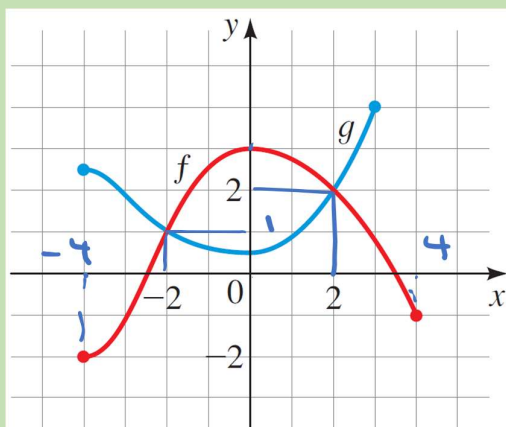
$$(b) g(-3) > f(-3)$$

$$(c) f(-2) = g(-2) = 1$$

$$f(2) = g(2) = 2$$

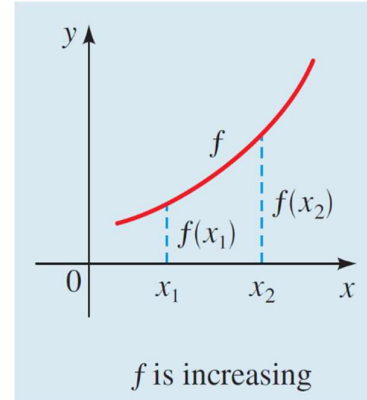
$$(d) x = [-4, -2] \cup [2, 4]$$

$$(e) x = (-2, 2)$$



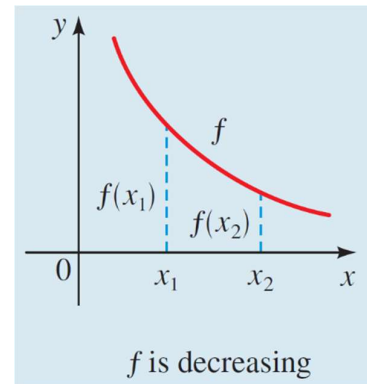
- A function is **increasing** متزايدة on an interval فترة I if $f(x_2) > f(x_1)$ for every $x_2 > x_1$
 إذا كانت قيمة الدالة $f(x_2)$ أكبر من $f(x_1)$ لكل قيم x_2 أكبر من x_1 في فترة معينة في الدالة، تسمى الدالة "متزايدة"

الدالة متزايدة لأن مع زيادة قيمة x تتزايد قيمة $f(x)$

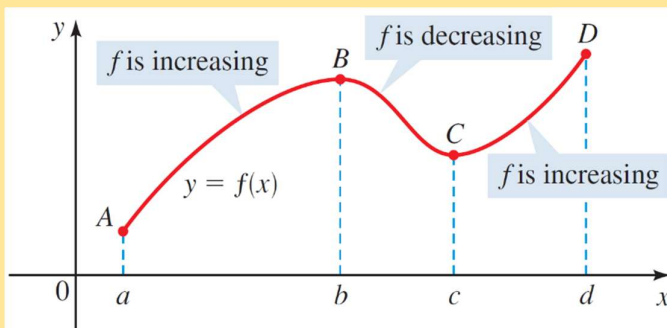


- A function is **decreasing** متناقصة on an interval فترة I if $f(x_2) < f(x_1)$ for every $x_2 > x_1$
 إذا كانت قيمة الدالة $f(x_2)$ أصغر من $f(x_1)$ لكل قيم x_2 أكبر من x_1 في فترة معينة في الدالة، تسمى الدالة "متناقصة"

الدالة متناقصة لأن مع زيادة قيمة x تتناقص قيمة $f(x)$



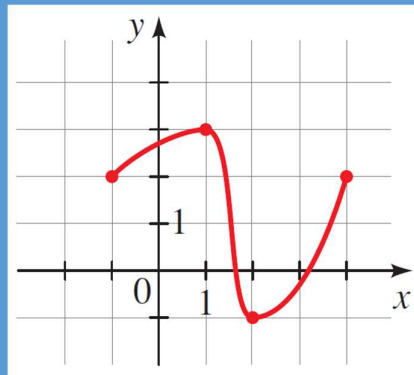
لاحظ



لتقرير إذا كانت الدالة متزايدة أو متناقصة
 نبدأ من اليمين إلى اليسار

Example 3

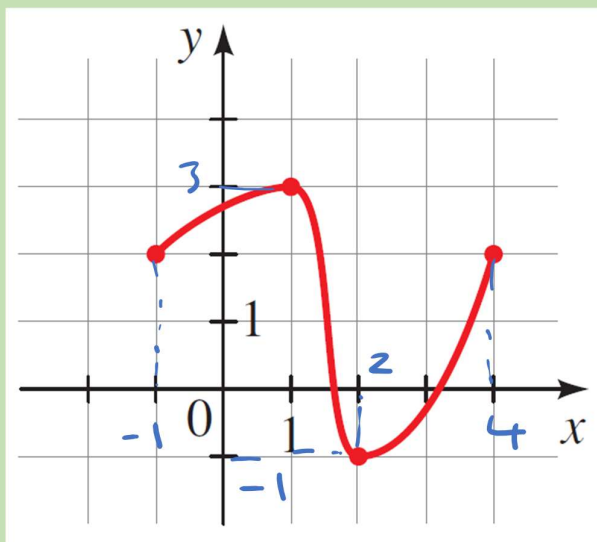
The graph of a function f is given. Use the graph to estimate the following. (a) The domain and range of f . (b) The intervals on which f is increasing and on which f is decreasing.

**Solution**

(a) $D: [-1, 4]$
 $R: [-1, 3]$

(b) Increasing:
 $(-1, 1)$ and $(2, 4)$

Decreasing:
 $(1, 2)$

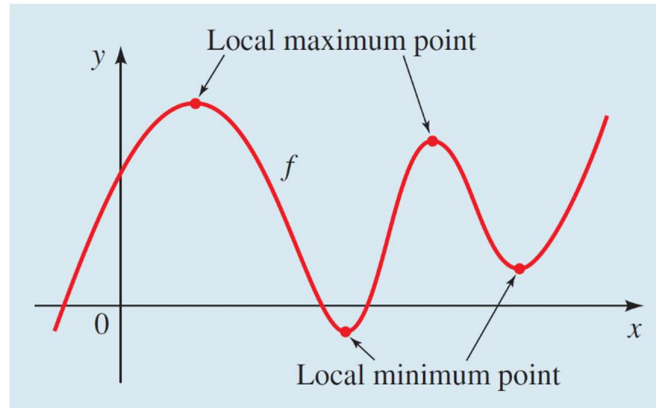


-The **Local maximum value** قيمة عظمى محلية is a value of function " $f(a)$ " such that

$$f(a) \geq f(x) \quad \longrightarrow \quad \begin{array}{l} f(a) \\ \text{أكبر من} \\ f(x) \end{array}$$

for all x in some open interval containing a .

In other words, f has local maximum at $x = a$.



-The **Local minimum value** قيمة صغرى محلية is a value of function " $f(a)$ " such that

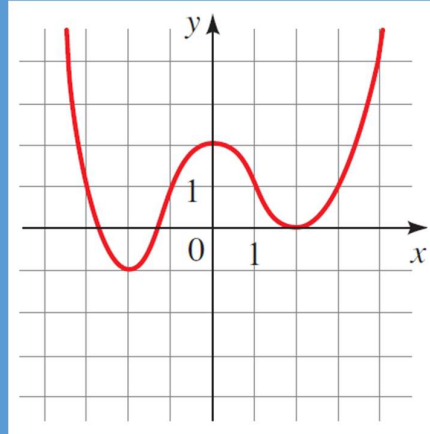
$$f(a) \leq f(x) \quad \longrightarrow \quad \begin{array}{l} f(a) \\ \text{أصغر من} \\ f(x) \end{array}$$

for all x in some open interval containing a .

In other words, f has local minimum at $x = a$.

Example 4

The graph of a function f is given. Use the graph to estimate the following. (a) All the local maximum and minimum values of the function and the value of x at which each occurs. (b) The intervals on which the function is increasing and on which the function is decreasing.



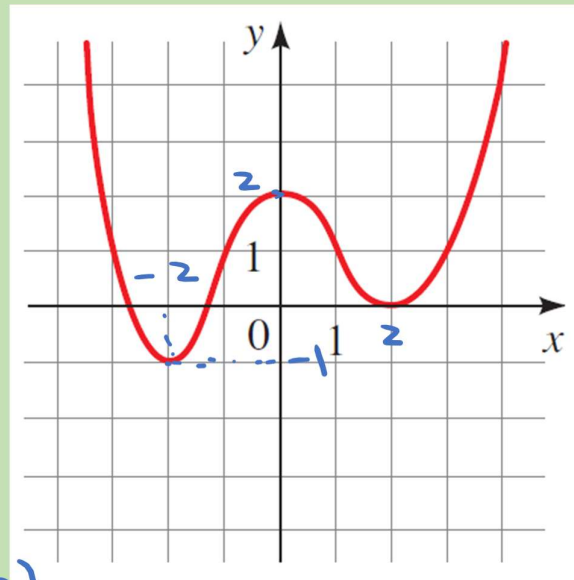
Solution

(a) $f(0) = 2$
is local max.

$f(-2) = -1$
and $f(2) = 0$
are local min.

(b) Increasing:
 $(-2, 0)$ and $(2, \infty)$

Decreasing: $(-\infty, -2)$ and $(0, 2)$



Problems

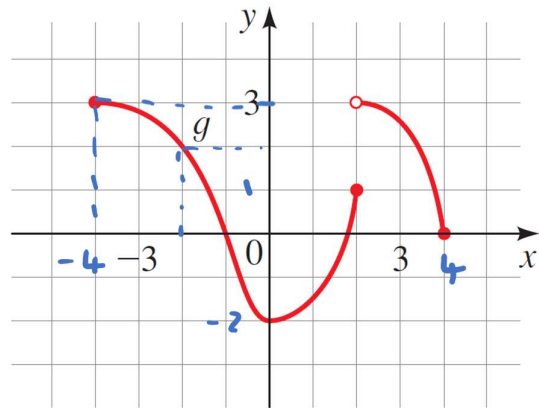
- The graph of a function g is given.

(a) Find $g(-4)$, $g(-2)$, $g(0)$, $g(2)$, and $g(4)$.

(b) Find the domain and range of g .

(c) Find the values of x for which $g(x) = 3$.

(d) Find the values of x for which $g(x) \leq 0$.



$$(a) \quad g(-4) = 3$$

$$g(-2) = 2$$

$$g(0) = -2$$

$$g(2) = 1$$

$$g(4) = 0$$

$$(b) \quad D: [-4, 4]$$

$$R: [-2, 3]$$

$$(c) \quad g(-4) = 3$$

$$(d) \quad x = [-1, 1.8]$$

- Graphs of the functions f and g are given.

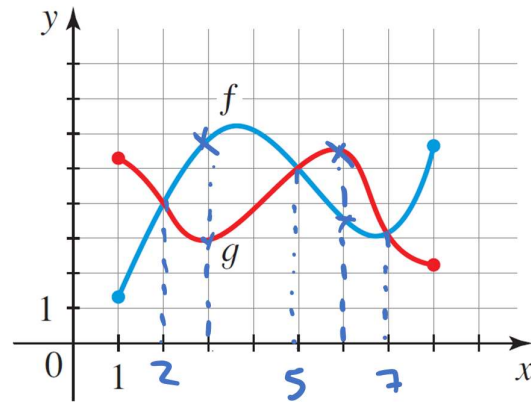
(a) Which is larger, $f(6)$ or $g(6)$?

(b) Which is larger, $f(3)$ or $g(3)$?

(c) For which values of x is $f(x) = g(x)$?

(d) Find the values of x for which $f(x) \leq g(x)$.

(e) Find the values of x for which $f(x) > g(x)$.



$$(a) \quad g(6) > f(6)$$

$$(b) \quad f(3) > g(3)$$

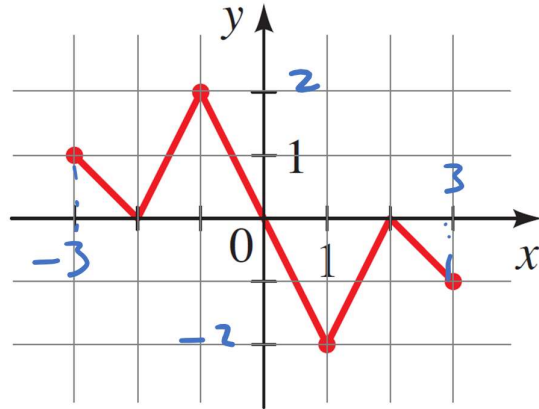
$$(c) \quad f(2) = g(2), \quad f(5) = g(5), \quad \text{and} \quad f(7) = g(7)$$

$$(d) \quad x = [1, 2] \cup [5, 7]$$

$$(e) \quad x = (2, 5) \cup (7, 8)$$

- The graph of a function f is given. Use the graph to estimate the following. (a) The domain and range of f .

(b) The intervals on which f is increasing and on which f is decreasing.



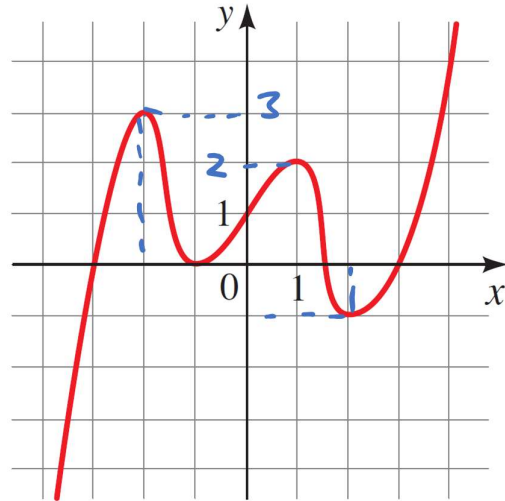
(a) $D: [-3, 3]$

$R: [-2, 2]$

(b) f is increasing on $(-2, -1)$ and $(1, 2)$

f is decreasing on $(-3, -2)$, $(-1, 1)$,
and $(2, 3)$

- The graph of a function f is given. Use the graph to estimate the following. (a) All the local maximum and minimum values of the function and the value of x at which each occurs. (b) The intervals on which the function is increasing and on which the function is decreasing.



$$(a) f(-2) = 3$$

$$\text{and } f(1) = 2$$

are local maxima

$$f(-1) = 0 \text{ and } f(2) = -1$$

are local minima

(b) f is increasing on

$(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$

f is decreasing on $(-2, -1)$ and $(1, 2)$