Section 2.7 – Combining Functions

- Two functions can be added, subtracted, multiplied, or divided

يمكن جمع، طرح، ضرب، و قسمة الدوال. و ينتج عنها دالة جديدة مداها domain هو تقاطع intersection مدى الدالتين $A \cap B$

Example 1

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

(a) Find the functions $f+g$, $f-g$, fg, and f/g and their domains.

(b) Find $(f+g)(4)$, $(f-g)(4)$, $(fg)(4)$, and $f/g(4)$

Solution

$f(x) = \frac{1}{x-2}$ $\quad D: (-\infty, 2) \cup (2, \infty)$

$g(x) = \sqrt{x}$ $\quad D: [0, \infty)$

(a) $f + g = \frac{1}{x-2} + \sqrt{x}$

$D: D(f(x)) \cap D(g(x))$

$D: [0, 2) \cup (2, \infty) = \{x \mid x \geq 0, x \neq 2\}$

$f - g = \frac{1}{x-2} - \sqrt{x}$ $\quad D: [0, 2) \cup (2, \infty)$

$fg = \frac{\sqrt{x}}{x-2}$ $\quad D: [0, 2) \cup (2, \infty)$

$f/g = \frac{1}{x-2} \div \sqrt{x} = \frac{1}{\sqrt{x}(x-2)}$

$D: (0, 2) \cup (2, \infty)$

(b) $f+g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{1}{2} + 2 = \frac{5}{2}$

- For the two functions f and g, the composite function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x))$$

الدالة المركبة تتكون من دالتين بحيث تستخدم مخرجات واحدة كمدخلات للدالة الأخرى

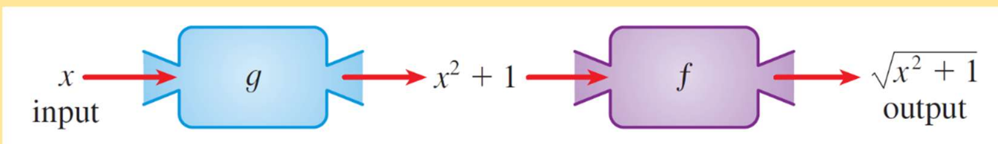
Example:

$f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$

$$f \circ g = f\big(g(x)\big) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

لاحظ

بدأنا من الدالة الداخلية في المثال $(g(x))$ ثم نستخدم الناتج منها كمدخل للدالة الخارجية $f(x)$



Example 2

Let $f(x) = x^2$ and $g(x) = x - 3$

(a) Find the functions $f \circ g$, and $g \circ f$ and their domains.

(b) Find $(f \circ g)(5)$, and $(g \circ f)(7)$

Solution

(a) $f \circ g = f(g(x)) = (x-3)^2$

$D: \mathbb{R}$

$g \circ f = g(f(x)) = x^2 - 3$

$D: \mathbb{R}$

(b) $(f \circ g)(5) = (5-3)^2 = 4$

$(g \circ f)(7) = 7^2 - 3 = 46$

Example 3

Use $f(x) = 2x - 3$ and $g(x) = 4 - x^2$ to evaluate the expression.

(a) $f(g(0))$

(b) $g(f(0))$

(c) $f(f(2))$

(d) $(f \circ g)(-2)$

Solution

(a) $g(0) = 4 - 0 = 4$

$f(g(0)) = f(4)$

$= 2(4) - 3 = 5$

(b) $f(0) = 2(0) - 3 = -3$

$g(f(0)) = 4 - (-3)^2$

$= 4 - 9 = -5$

(c) $f(2) = 2(2) - 3$

$= 4 - 3 = 1$

$f(f(2)) = 2(1) - 3 = -1$

(d) $(f \circ g)(-2) = f(g(-2))$

$g(-2) = 4 - (-2)^2 = 4 - 4 = 0$

$f(g(-2)) = 2(0) - 3 = -3$

Example 4

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find the following functions and their domains

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

Solution

(a) $f \circ g = f(g(x))$

$= f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$

$2 - x \geq 0$

$-x \geq -2$

$x \leq 2$

$D: (-\infty, 2]$

(b) $g \circ f = g(f(x))$

$= g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$

$2 - \sqrt{x} \geq 0$ and $x \geq 0$

$-\sqrt{x} \geq -2$

$\sqrt{x} \leq 2$

$x \leq 4$

$D: [0, 4]$

(c) $f \circ f = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}}$

$D: [0, \infty) = \{x \mid x \geq 0\}$

Problems

- Find $f+g$, $f-g$, fg, and f/g and their domains.

(a) $f(x)=x^2+x$, $g(x)=x^2$

(b) $f(x)=5-x$, $g(x)=x^2-3x$

(a) $f+g = x^2+x+x^2 = 2x^2+x$ $D:\mathbb{R}$

$f-g = x^2+x-x^2 = x$ $D:\mathbb{R}$

$fg = (x^2+x)(x^2) = x^4+x^3$ $D:\mathbb{R}$

$f/g = \dfrac{x^2+x}{x^2}$ $D:\mathbb{R}/\{0\}$

(b) $f+g = 5-x+x^2-3x = x^2-4x+5$ $D:\mathbb{R}$

$f-g = 5-x-(x^2-3x)$

$= 5-x-x^2+3x = 5+2x-x^2$ $D:\mathbb{R}$

$fg = (5-x)(x^2-3x)$

$= 5x^2-15x-x^3+3x^2 = 8x^2-15x-x^3$

$D:\mathbb{R}$

$f/g = \dfrac{5-x}{x^2-3x}$

$x^2-3x=0 \Rightarrow x(x-3)=0$

$x=0$ or $x=3$

$D:\mathbb{R}/\{0,3\}$

- Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$f(x) = x^2,\ g(x) = x + 1$

$$f \circ g = f(g(x))$$
$$= (x+1)^2 = x^2 + 2x + 1$$
$D: \mathbb{R}$

$$g \circ f = g(f(x))$$
$$= x^2 + 1$$
$D: \mathbb{R}$

$$f \circ f = f(f(x))$$
$$= (x^2)^2 = x^4$$
$D: \mathbb{R}$

$$g \circ g = g(g(x))$$
$$= (x+1) + 1 = x + 2$$
$D: \mathbb{R}$

- Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$f(x) = \frac{x}{x+1}$, $g(x) = 2x - 1$

$$f \circ g = f(g(x))$$

$$= \frac{2x-1}{(2x-1)+1} = \frac{2x-1}{2x} \qquad D: \mathbb{R}/\{0\}$$

$$g \circ f = g(f(x))$$

$$= 2\left(\frac{x}{x+1}\right) - 1 = \frac{2x}{x+1} - 1$$

$$= \frac{2x-(x+1)}{x+1} = \frac{x-1}{x+1} \qquad D: \mathbb{R}/\{-1\}$$

$$f \circ f = \frac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \left(\frac{x}{x+1}\right) \div \left(\frac{x}{x+1}+1\right)$$

$$= \frac{x}{x+1} \div \left(\frac{x+x+1}{x+1}\right) = \frac{x}{x+1} \div \frac{2x+1}{x+1}$$

$$= \frac{x}{\cancel{x+1}} \cdot \frac{\cancel{x+1}}{2x+1} = \frac{x}{2x+1}$$

$2x+1=0$

$x = -\frac{1}{2}$

$D: \mathbb{R}/\{-\frac{1}{2}\}$

$$g \circ g = g(g(x))$$

$$= 2(2x-1)-1 = 4x-2-1$$

$$= 4x-3 \qquad D: \mathbb{R}$$

- Use $f(x) = 5 - x$ and $g(x) = x^2 - 3x$ to evaluate the expression.

(a) $g(f(-2))$

(b) $f(g(3))$

(c) $(f \circ f)(-1)$

(d) $(g \circ g)(1)$

(a) $f(-2) = 5 - (-2) = 5 + 2 = 7$

$g(f(-2)) = g(7)$

$= 7^2 - 3(7) = 49 - 21 = 28$

(b) $g(3) = 3^2 - 3(3) = 0$

$f(g(3)) = f(0) = 5 - 0 = 5$

(c) $f(-1) = 5 - (-1) = 5 + 1 = 6$

$f(f(-1)) = 5 - 6 = -1$

(d) $g(1) = 1^2 - 3(1) = -2$

$g(g(1)) = (-2)^2 - 3(-2)$

$= 4 + 6 = 10$

- Use the table to evaluate the expression.

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

(a) $f(g(2))$

(b) $(f \circ g)(6)$

(c) $(f \circ f)(5)$

(d) $f(5) - 3g(6)$

(a) $g(2) = 5$

$f(g(2)) = f(5) = 6$

(b) $g(6) = 4$

$f(g(6)) = f(4) = 1$

(c) $f(5) = 1$

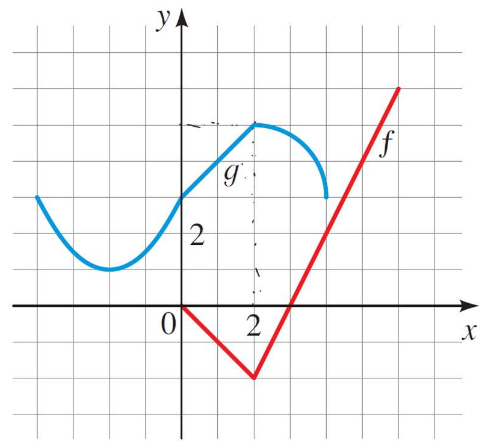
$f(f(5)) = f(1) = 3$

(d) $f(5) - 3g(6)$

$= 6 - 3(4)$

$= 6 - 12 = -6$

- Use the given graphs of f and g to evaluate the expression.



(a) $f(g(2))$

(b) $(g \circ f)(4)$

(c) $(g \circ g)(-2)$

(a) $g(2) = 5$

$f(5) = 4$

(b) $f(4) = 2$

$g(2) = 5$

(c) $g(-2) = 1$

$g(1) = 4$