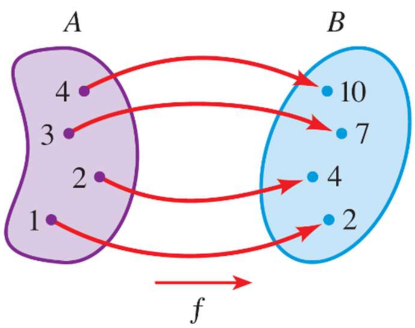
Section 2.8 – One-to-One Functions and Their Inverses

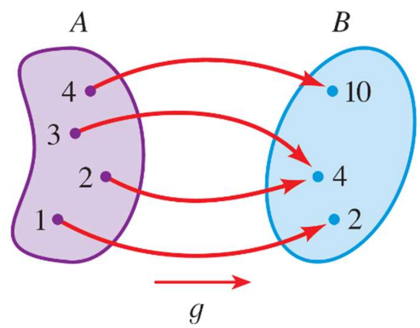
- A function with each element عنصر in the range matches one, and-only-one, element in the domain is called one-to-one function

الدالة التي يرتبط كل عنصر من المجال بعنصر واحد فقط من المدى تسمى دالة متباينة



One-to-one function

لأن كل عنصر من المجال range مرتبط بعنصر واحد فقط من المدى domain

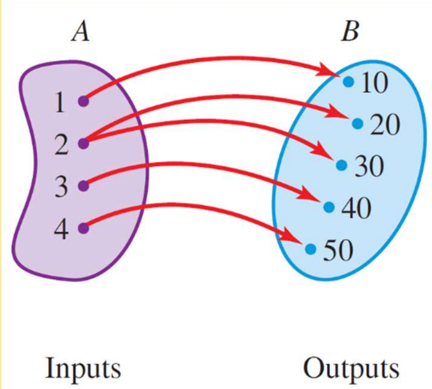


Not one-to-one function

لأن أحد عناصر المجال range مرتبط بأكثر من عنصر من المدى domain

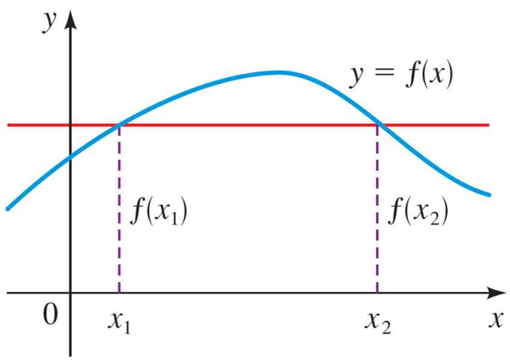
لاحظ

تذكر أن ارتباط أحد عناصر المدى domain بأكثر من عنصرفي المجال range يعني أن العلاقة ليست دالة أصلا



(b) Not a function

- يمكن معرفة إذا كانت الدالة one-to-one من الرسم، برسم خط أفقي: إذا قطع منحنى الدالة أكثر من مرة لا تكون one-to-one



Not one-to-one function

لأن الخط الأفقي قطع منحنى الدالة مرتين

Example 1

Is the function $f(x) = x^2$ one-to-one?

Solution

$f(a) = f(b)$

$a^2 = b^2$

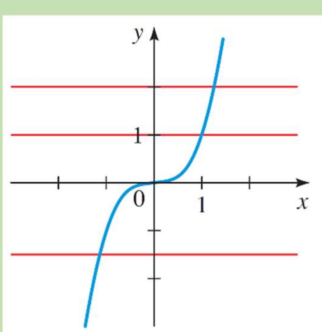
$a = \pm\sqrt{b^2}$

$a = \pm b$ Not one-to-one

Example 2

Is the function $f(x) = x^3$ one-to-one?

Solution



$f(a) = f(b)$

$a^3 = b^3$

$a = b$ One-to-one

In order for $f(a)$ to be equal to $f(b)$ a must be equal to b

Example 3

Show that the function $f(x) = 3x + 4$ is one-to-one.

Solution

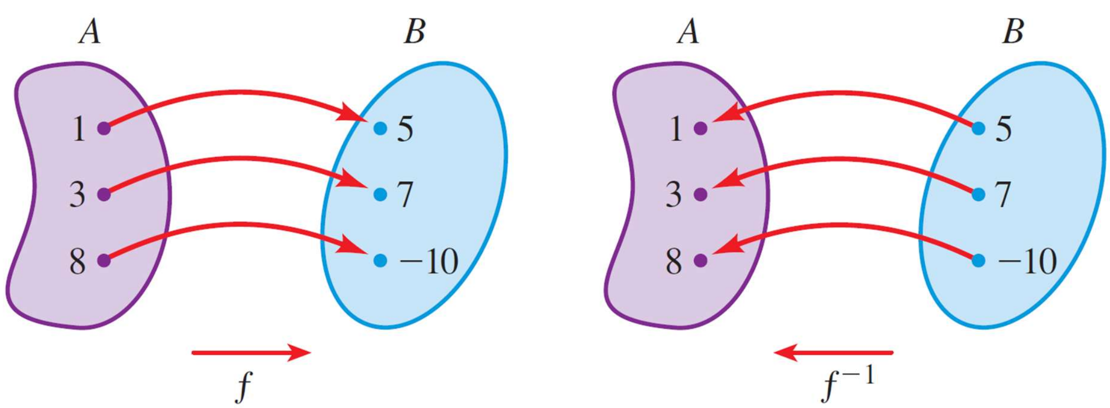
$f(a) = f(b)$

$3a + \cancel{4} = 3b + \cancel{4}$

$\cancel{3}a = \cancel{3}b$

$a = b$ One-to-one

- For any one-to-one function $f(x) = y$ with domain A and range B, there is an inverse function دالة عكسية $f^{-1}(y) = x$ with domain B and range A



Example 4

If $f(1) = 5$, and $f(8) = -10$, find $f^{-1}(5)$, and $f^{-1} = (-10)$

Solution

$f^{-1}(5) = 1$

$f^{-1}(-10) = 8$

Example 5

Given $f(x) = \sqrt{x-2}$. Find $f^{-1}(5)$

Solution

$f(x) = 5$

$\sqrt{x-2} = 5$

$x - 2 = 25$

$x = 27$

$f^{-1}(5) = 27$

- To **find the inverse of a function**

1- اكتب $y = f(x)$

2- أوجد x كمعادلة في y

3- بدل x و y لتصبح المعادلة $y = f^{-1}(x)$

Example 6

Find the inverse of the functions.

(a) $f(x) = 3x - 2$

(b) $f(x) = \frac{x^5-3}{2}$

Solution

(a) $y = 3x - 2$

$y + 2 = 3x$

$\frac{y+2}{3} = x$

$x = \frac{y+2}{3}$

$f^{-1}(x) = \frac{x+2}{3}$

(b) $y = \frac{x^5-3}{2}$

$2y = x^5 - 3$

$2y + 3 = x^5$

$x = \sqrt[5]{2y+3}$

$f^{-1}(x) = \sqrt[5]{2x+3}$

- **Inverse Function Property خاصية الدالة العكسية:**

for any function $f(x)$ with an inverse function $f^{-1}(x)$ ➔ $f\left(f^{-1}(x)\right) = f^{-1}\left(f(x)\right) = x$

مثال:

$f(x) = 3x - 2$ and $f^{-1}(x) = \frac{x+2}{3}$

$$\therefore f(f^{-1}(x)) = 3\frac{x+2}{3} - 2 = x$$

Example 7

Use the Inverse Function Property to show that f and g are inverses of each other.

$f(x) = 2 - 5x,\ g(x) = \frac{2-x}{5}$

Solution

$$f(g(x)) = 2 - \cancel{5}\left(\frac{2-x}{\cancel{5}}\right)$$

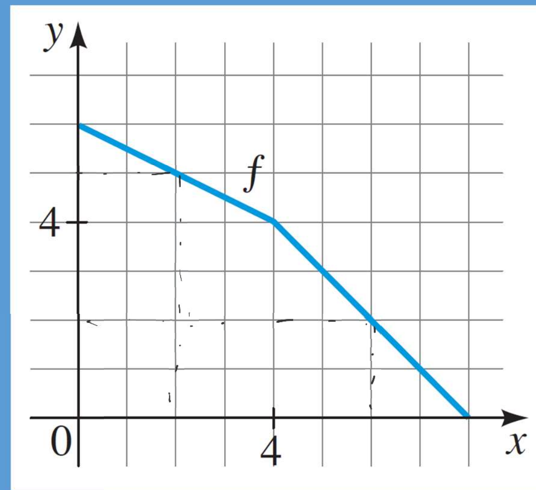
$$= 2 - (2 - x)$$

$$= \cancel{2} - \cancel{2} + x = x$$

Therefore $f(x)$ and $g(x)$ are inverse

Example 8

A graph of a function is given.
Use the graph to find the indicated values.
(a) $f^{-1}(2)$
(b) $f^{-1}(5)$
(c) $f^{-1}(6)$



Solution

(a) $f^{-1}(2) = 6$

(b) $f^{-1}(5) = 2$

(c) $f^{-1}(6) = 0$

Example 9

A table of values for a one-to-one function is given. Find (a) $f^{-1}(2)$, (b) $f^{-1}(0)$

x	1	2	3	4	5	6
$f(x)$	4	6	2	5	0	1

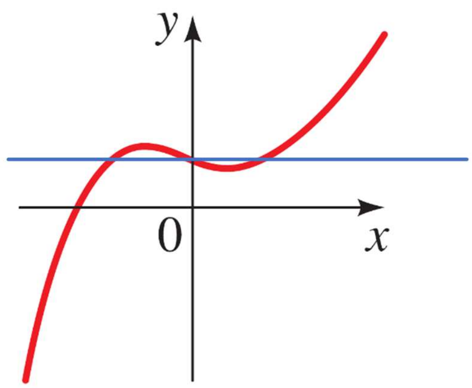
Solution

(a) $f^{-1}(2) = 3$

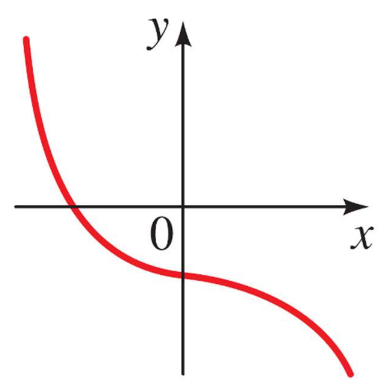
(b) $f^{-1}(0) = 5$

Problems

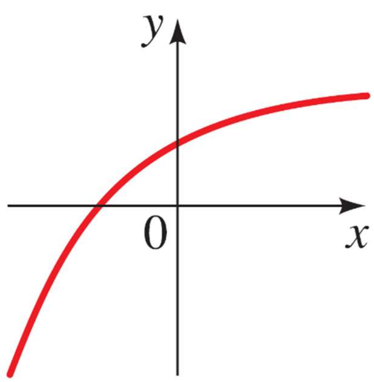
- A graph of a function f is given. Determine whether f is one-to-one.



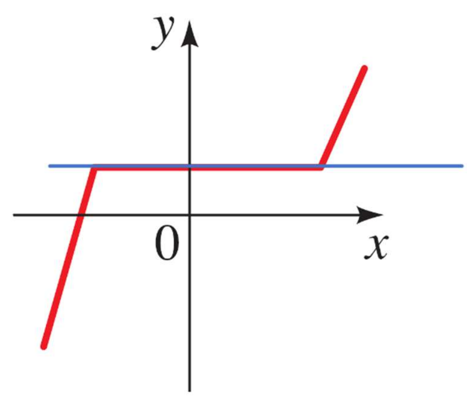
Not one-to-one



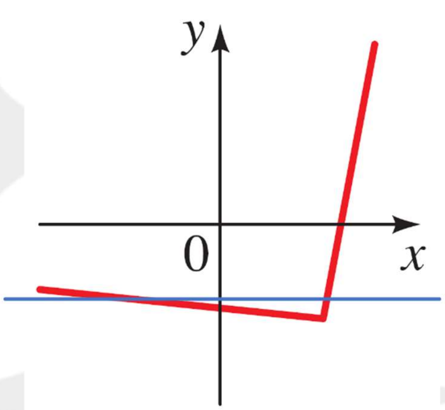
One-to-one



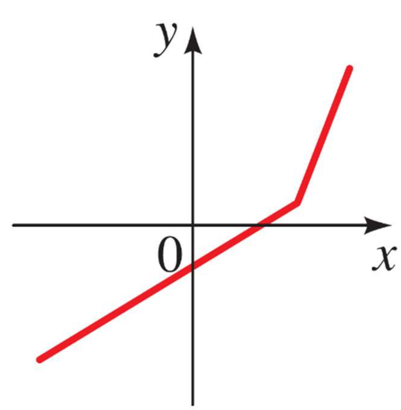
One-to-one



Not one-to-one



Not one-to-one



One-to-one

- Determine whether the function is one-to-one function.

(a) $f(x) = -2x + 4$

$$-2x_1 + \cancel{4} = -2x_2 + \cancel{4}$$

$$\cancel{-2}x_1 = \cancel{-2}x_2$$

$$x_1 = x_2$$

$\therefore$ One-to-one

(b) $g(x) = \sqrt{x}$

$$(\sqrt{x_1})^2 = (\sqrt{x_2})^2$$

$$x_1 = x_2$$

$\therefore$ One-to-one

(c) $h(x) = x^2 - 2x$

$$h(0) = 0 - 2(0) = 0$$

$$h(2) = 2^2 - 2(2) = 4 - 4 = 0$$

$\because$ $h(0) = h(2)$

$\therefore$ Not one-to-one

- Find the inverse function of f.

(a) $f(x) = 3x + 5$

$$y = 3x + 5$$

$$y - 5 = 3x$$

$$\frac{y-5}{3} = x$$

$$f^{-1}(x) = \frac{x-5}{3}$$

(b) $f(x) = \frac{x}{x+4}$

$$y = \frac{x}{x+4}$$

$$y(x+4) = x$$

$$yx + 4y - x = 0$$

$$yx - x = -4y$$

$$x(y-1) = -4y$$

$$x = \frac{-4y}{y-1}$$

$$f^{-1}(x) = \frac{-4x}{x-1}$$

(c) $f(x) = 4 - x$

$$y = 4 - x$$

$$y - 4 = -x$$

$$x = -y + 4$$

$$f^{-1}(x) = -x + 4$$

- (a) Find the inverse function of the function $f(x) = \frac{2x+3}{x-1}$

$$y = \frac{2x+3}{x-1}$$

$$y(x-1) = 2x+3$$

$$yx - y = 2x + 3$$

$$yx - 2x = 3 + y$$

$$x(y-2) = y+3$$

$$x = \frac{y+3}{y-2}$$

$$f^{-1}(x) = \frac{x+3}{x-2}$$

(b) Find the domain and range of $f(x)$

$$x - 1 = 0$$

$$x = 1$$

$$D: \mathbb{R}/\{1\}$$

To find the range we find the domain of the inverse

$$x - 2 = 0$$

$$x = 2 \quad \Rightarrow \quad R: \mathbb{R}/\{2\}$$

(c) Find the domain and range of $f^{-1}(x)$

$$D: f^{-1}(x) = R: f(x) = \mathbb{R}/\{2\}$$

$$R: f^{-1}(x) = D: f(x) = \mathbb{R}/\{1\}$$