

Section 3.3 – Dividing Polynomials

- If $P(x)$ and $D(x)$ are polynomials دوال كثيرة حدود, and $D(x) \neq 0$, then we can divide نقسم $P(x)$ by $D(x)$ such that:

$$\begin{array}{c} \text{Dividend} \\ \frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)} \end{array} \quad \begin{array}{c} \text{Quotient} \\ \text{Remainder} \end{array} \quad \Longrightarrow \quad P(x) = D(x) \cdot Q(x) + R(x)$$

Divisor

يمكن قسمة دالة كثيرة حدود على دالة كثيرة حدود بنفس طريق قسمة رقم على رقم، و تسمى هذه الطريقة:

القسمة المطولة long division

لاحظ

طريقة قسمة الدوال نفس الأرقام

$$\begin{array}{c} \text{Dividend} \\ \frac{38}{7} = 5 + \frac{3}{7} \end{array} \quad \begin{array}{c} \text{Quotient} \\ \text{Remainder} \end{array}$$

Divisor

Example 1

Divide $6x^2 - 26x + 12$ by $x - 4$

Solution

$$\begin{array}{r} 6x - 2 \\ x-4 \overline{) 6x^2 - 26x + 12} \\ \underline{- 6x^2 + 24x} \\ -2x + 12 \\ \underline{+ 2x - 8} \\ 4 \end{array} \quad \left| \quad \frac{6x^2 - 26x + 12}{x - 4} = 6x - 2 + \frac{4}{x - 4} \right.$$

Example 2

Let $P(x) = 8x^4 + 6x^2 - 3x + 1$ and $D(x) = 2x^2 - x + 2$. Divide $P(x)$ by $D(x)$.

Solution

$$\begin{array}{r}
 4x^2 + 2x \\
 2x^2 - x + 2 \overline{) 8x^4 + 6x^2 - 3x + 1} \\
 \underline{8x^4 - 4x^3 + 8x^2} \\
 4x^3 - 2x^2 - 3x + 1 \\
 \underline{4x^3 - 2x^2 + 4x} \\
 -7x + 1
 \end{array}$$

$$\frac{8x^4 + 6x^2 - 3x + 1}{2x^2 - x + 2} = 4x^2 + 2x + \frac{-7x + 1}{2x^2 - x + 2}$$

- **Synthetic division** القسمة التركيبية

نقدر نستخدمها **فقط** إذا كان المقسوم عليه على صورة $x - c$ (جمع أو طرح)

Example 3

Use synthetic division to divide $2x^3 - 7x^2 + 5$ by $x - 3$

Solution

Synthetic division

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & \downarrow & & & \\ & 6 & -3 & -9 & \\ \hline & 2 & -1 & -3 & -4 \end{array}$$

Long division

$$\begin{array}{r} 2x^2 - x - 3 \\ x-3 \overline{) 2x^3 - 7x^2 + 0x + 5} \\ \underline{2x^3 - 6x^2} \\ -x^2 + 0x + 5 \\ \underline{-x^2 + 3x} \\ -3x + 5 \\ \underline{-3x + 9} \\ -4 \end{array}$$

$$\frac{2x^3 - 7x^2 + 5}{x - 3} = 2x^2 - x - 3 - \frac{4}{x - 3}$$

- Remainder Theorem:

إذا كان المقسوم عليه على صورة $x - c$ فإن متبقي القسمة يساوي $P(c)$

Example 4

Find the remainder of dividing $P(x) = x^3 + 3x^2 - 7x + 6$ by $x - 2$

Solution

$$\begin{aligned} \text{remainder} = P(2) &= 2^3 + 3(2)^2 - 7 \cdot 2 + 6 \\ &= 8 + 12 - 14 + 6 \\ &= 12 \end{aligned}$$

Another sol.

$$\begin{array}{r} x^2 + 5x + 3 \\ x-2 \overline{) x^3 + 3x^2 - 7x + 6} \\ \underline{-x^3 + 2x^2} \\ 5x^2 - 7x + 6 \\ \underline{-5x^2 + 10x} \\ 3x + 6 \\ \underline{-3x + 6} \\ 12 \end{array}$$

Example 5

Find the remainder of dividing $P(x) = x^3 + 3x^2 - 7x + 6$ by $x + 2$

Solution

$$\begin{aligned} P(-2) &= (-2)^3 + 3(-2)^2 - 7(-2) + 6 \\ &= -8 + 12 + 14 + 6 \\ &= 24 \end{aligned}$$

- **Factor Theorem:** c is a zero of P if, and only if, $x - c$ is a factor of $P(x)$

إذا كانت $P(c) = 0$ فإن c هي أحد أصفار الدالة $P(x)$ وبالتالي فإن $x - c$ هي أحد معاملات $P(x)$ (factors)
أي أن $P(x)$ تقبل القسمة على $x - c$ بدون متبقي remainder

Example 6

Which of the following is a factor of $P(x) = x^3 - 7x - 6$?

(a) $x + 2$

(b) $x - 1$

(c) $x + 4$

(d) $x + 3$

Solution

(a) $P(-2) = (-2)^3 - 7(-2) - 6 = -8 + 14 - 6 = 0$

(b) $P(1) = 1^3 - 7 \cdot 1 - 6 = -12$

(c) $P(-4) = (-4)^3 - 7(-4) - 6 = -42$

(d) $P(-3) = (-3)^3 - 7(-3) - 6 = -12$

Example 7

Let $P(x) = x^3 - 7x + 6$. Show that 1 is a zero of $P(x)$, and use this fact to find all other real zeros.

Solution

$P(1) = 1^3 - 7(1) + 6 = 0$
 $\therefore 1$ is a zero of $P(x)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$P(x) = (x - 1)(x^2 + x - 6)$
 $= (x - 1)(x + 3)(x - 2)$

zeros of $P(x)$
are: $\{1, -3, 2\}$

Problems

- Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$

(a) $P(x) = -x^3 - 2x + 6, D(x) = x + 1$

(b) $P(x) = 8x^4 + 4x^3 + 6x^2, D(x) = 2x^2 + 1$

- Find the quotient and remainder using long division

$$\frac{x^3 + 2x + 1}{x^2 - x + 3}$$

- Find the quotient and remainder using synthetic division

$$\frac{x^3 - 8x + 2}{x + 3}$$

- Use synthetic division and the Remainder Theorem to evaluate $P(c)$

$$P(x) = 4x^2 + 12x + 5, c = -1$$

- Use the Factor Theorem to show that $x - c$ is a factor of $P(x)$ for the given value of c .

$$P(x) = x^3 - 3x^2 + 3x - 1, c = 1$$

-Show that the given value of c is a zero of $P(x)$, and find all other zeros of $P(x)$

$$P(x) = x^3 + 2x^2 - 9x - 18, c = -2$$