Section 4.4 – Laws of Logarithms

- Laws of Logarithms قوانين اللوغاريتمات

$\log_a A + \log_a B = \log_a(AB)$ $\quad\quad$ $\ln A + \ln B = \ln(AB)$

$\log_a A - \log_a B = \log_a\left(\frac{A}{B}\right)$ $\quad\quad$ $\ln A - \ln B = \ln\left(\frac{A}{B}\right)$

$\log_a(A^C) = C\log_a A$ $\quad\quad$ $\ln(A^C) = C\ln A$

لاحظ

Common MISTAKES

1. $\log_a(A+B) \neq \log_a A + \log_a B$
2. $\frac{\log_a A}{\log_a B} \neq \log_a\left(\frac{A}{B}\right)$
3. $(\log_a A)^C \neq C\log_a A$

Example 1

Evaluate each expression.
(a) $\log_4 2 + \log_4 32$
(b) $\log_2 80 - \log_2 5$
(c) $\log_3 100 - \log_3 18 - \log_3 50$

Solution

(a) $\log_4(2\cdot 32) = \log_4 64 = 3$

(b) $\log_2\left(\frac{80}{5}\right) = \log_2 16 = 4$

(c) $\log_3\left(\frac{100}{50}\right) - \log_3 18 = \log_3 2 - \log_3 18$

$= \log_3\left(\frac{2}{18}\right) = \log_3 \frac{1}{9} = -2$

Problems

- Evaluate the expression

(a) $\frac{-1}{3}\log 8$

$= \log\left(8^{-1/3}\right)$

$= \log\left(\frac{1}{8^{1/3}}\right)$

$= \log \frac{1}{\sqrt[3]{8}} = \log \frac{1}{2}$

(b) $\log 50 + \log 200$

$= \log (50 \cdot 200)$

$= \log 10{,}000 = 4$

(c) $\log_2 60 - \log_2 15$

$= \log_2 \left(\frac{60}{15}\right)$

$= \log_2 4 = 2$

- Use the Laws of Logarithms to expand the expression.

(a) $\log_5(5x^3y^6)$

$= \log_5 5 + \log_5 x^3 + \log_5 y^6$

$= 1 + 3\log_5 x + 6\log_5 y$

(b) $\ln\left(\frac{ab^2}{\sqrt[3]{c}}\right)$

$= \ln ab^2 - \ln \sqrt[3]{c}$

$= \ln a + \ln b^2 - \ln c^{1/3}$

$= \ln a + 2\ln b - \frac{1}{3}\ln c$

(c) $\log \sqrt{\frac{x}{10(x^3-7)^2}}$

$= \log \left(\frac{x}{10(x^3-7)^2}\right)^{1/2}$

$= \frac{1}{2} \log \frac{x}{10(x^3-7)^2}$

$= \frac{1}{2}\left[\log x - \log 10(x^3-7)^3\right]$

$= \frac{1}{2}\left[\log x - \log 10 + \log (x^3-7)^3\right]$

$= \frac{1}{2}\left[\log x - \log 10 + 3\log (x^3-7)\right]$

(d) $\ln\left(x\sqrt{\frac{y}{z}}\right)$

$= \ln x + \ln \sqrt{\frac{y}{z}}$

$= \ln x + \ln \left(\frac{y}{z}\right)^{1/2}$

$= \ln x + \frac{1}{2} \ln \left(\frac{y}{z}\right)$

$= \ln x + \frac{1}{2}\left[\ln y - \ln z\right]$

- Use the Laws of Logarithms to combine the expression.

(a) $1 + 3\log x + \frac{1}{2}\log(x+1)$

$= \log 10 + \log x^3 + \log (x+1)^{1/2}$

$= \log [10 x^3 \sqrt{x+1}]$

(b) $3\ln s + \ln(t+2) - \ln(t^2+t-2)$

$= \ln s^3 + \ln(t+2) - \ln(t^2+t-2)$

$= \ln \frac{s^3(t+2)}{t^2+t-2}$

(c) $\ln(a+b)+\ln(a-b)-2\ln c$

$$= \ln \frac{(a+b)(a-b)}{c^2}$$

$$= \ln \frac{a^2-b^2}{c^2}$$

(d) $2\log_3 x - 3(\log_3(y+1)+\log_3 z)$

$$= \log_3 x^2 - 3\log_3(y+1) + 3\log_3 z$$

$$= \log_3 x^2 - \log_3 (y+1)^3 + \log_3 z^3$$

$$= \log_3 \frac{x^2 z^3}{(y+1)^3}$$