

Section 4.5 – Exponential and Logarithmic Equations

- المعادلة الأسية exponential equation هي معادلة يكون فيها المتغير في الأس $5^x \leftarrow$

Example 1

Solve the exponential equation.

(b) $5^{2x} = 5^{x+1}$

(b) $3^{x+2} = 7$

Solution

$$\begin{aligned} (a) \quad 2x &= x+1 \\ 2x - x &= 1 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} (b) \quad \ln 3^{x+2} &= \ln 7 \\ x+2 \ln 3 &= \ln 7 \\ x+2 &= \frac{\ln 7}{\ln 3} \\ x &= \frac{\ln 7}{\ln 3} - 2 \end{aligned}$$

- المعادلة اللوغاريتمية logarithmic equation هي معادلة المتغير فيها داخل لوغاريتم $\log_a x \leftarrow$

Example 2

Solve the equation $\log(x^2 + 1) = \log(x - 2) + \log(x + 3)$

Solution

$$\log(x^2 + 1) = \log(x - 2)(x + 3)$$

$$x^2 + 1 = (x - 2)(x + 3)$$

$$x^2 + 1 = x^2 + x - 2x - 6$$

$$\cancel{x^2} - \cancel{x^2} - x = -6 - 1$$

$$-x = -7 \Rightarrow x = 7$$

Problems

- Solve the equation.

(a) $5^x = 125$

$$5^x = 5^3$$

$$x = 3$$

(b) $10^{2x-1} = \frac{1}{10^{6+5x}}$

$$10^{2x-1} = 10^{-(6+5x)}$$

$$2x - 1 = -6 - 5x$$

$$2x + 5x = -6 + 1$$

$$7x = -5$$

$$x = -\frac{5}{7}$$

(c) $3 + 2e^{3-2x} = 11$

$$2e^{3-2x} = 11 - 3$$

$$e^{3-2x} = \frac{8}{2}$$

$$\ln e^{3-2x} = \ln 4$$

$$3 - 2x \cdot \ln e = \ln 4$$

$$3 - 2x = \ln 4$$

$$-2x = \ln 4 - 3$$

$$x = \frac{\ln 4 - 3}{-2}$$

(d) $e^{2x} - e^x - 6 = 0$

$$(e^x)^2 - e^x - 6 = 0$$

$$(e^x - 3)(e^x + 2) = 0$$

$$e^x = 3 \quad \text{or} \quad e^x = -2$$

$$\ln e^x = \ln 3$$

$$x \cdot \ln e = \ln 3$$

$$x = \ln 3$$

$$\ln e^x = \ln(-2)$$

X
rejected

(e) $3^{2x} - 4(3^x) + 3 = 0$

$$(3^x)^2 - 4(3^x) + 3 = 0$$

$$(3^x - 3)(3^x - 1) = 0$$

$$3^x - 3 = 0$$

or

$$3^x - 1 = 0$$

$$3^x = 3$$

$$3^x = 1$$

$$x = 1$$

$$x = 0$$

(f) $3xe^x + x^2e^x = 0$

$$xe^x(3 + x) = 0$$

e^x cannot equal zero

$$x = 0$$

or

$$3 + x = 0$$

$$x = -3$$

(g) $\ln x = 8$

$$x = e^8$$

(h) $\log_2(25 - x) = 3$

$$25 - x = 2^3$$

$$25 - x = 8$$

$$-x = 8 - 25 = -17$$

$$x = 17$$

(i) $4 + 3 \log(2x) = 16$

$$3 \log(2x) = 16 - 4$$

$$\log(2x)^3 = 12$$

$$\sqrt[3]{(2x)^3} = \sqrt[3]{10^{12}}$$

$$2x = 10^4$$

$$2x = 10,000$$

$$x = \frac{10,000}{2} = 5,000$$

(j) $\log_5 x + \log_5(x+2) = \log_5 3$

$$\log_5 x(x+2) = \log_5 3$$

$$x(x+2) = 3$$

$$x^2 + 2x = 3$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3$$

or

$$x = 1$$

X
rejected