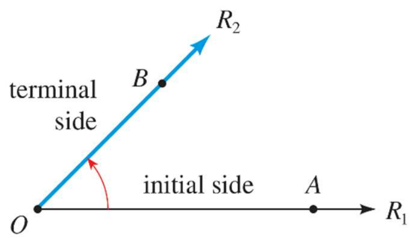
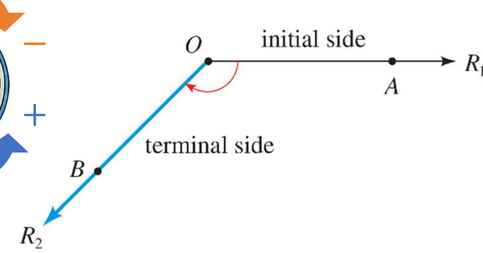
# Section 6.1 – Angle Measure

- أي زاوية لها شعاعين (خطين) ونقطة تلاقي vertex تسمى رأس الزاوية.

إذا كان دوران الزاوية عكس عقارب الساعة counterclockwise يكون قياسها موجب، وسالب إذا كان الدوران مع عقارب الساعة clockwise.




Positive angle

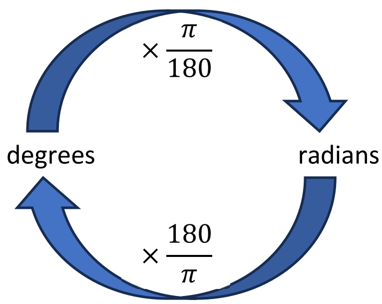

Negative angle

الزاوية التي تدور عكس عقارب الساعة موجبة

الزاوية التي تدور مع عقارب الساعة سالبة

- تقاس الزوايا بالدرجة **degrees** أو راديان **radians**

ونقدر نحول بينهم كما بالشكل ←




## Example 1

(a) Express 60° in radians.

(b) Express $\frac{\pi}{6}$ rad in degrees.

**Solution**

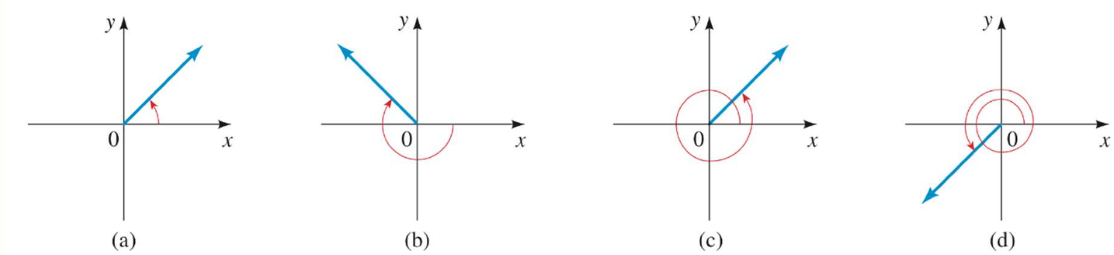
(a) $60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ rad

(b) $\frac{\pi}{6}$ rad $= \frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$

**لاحظ**

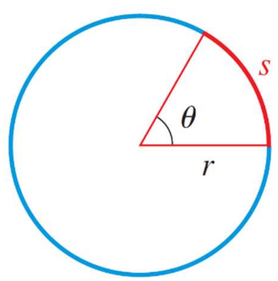
نقول أن الزاوية في الوضع الطبيعي **standard position** إذا كانت رأسها عند نقطة الأصل origin و خط البداية هو محور $x$

**Examples:**




- طول القوس $s$ المقابل للزاوية $\theta$ في مركز دائرة نصف قطرها $r$ يمكن حسابه من المعادلة $s = r\theta$

لازم تكون radians




## Example 2

Find the length of the circular arc.

(a) $\theta = \frac{5\pi}{14}$ and $r = 7$

(b) $\theta = 30°$ and $r = 10$

**Solution**

(a) $s = r\theta = 7 \frac{5\pi}{14} = \frac{5}{2}\pi$ m

(b) $30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$

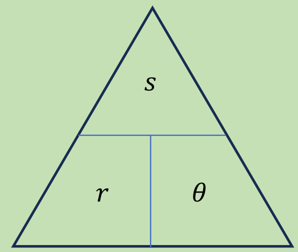
$s = r\theta = 10 \frac{\pi}{6} = \frac{5}{3}\pi$ m

## Example 3

Find the measure of the central angle $\theta$ that has a circular arc $s$ of length 6 m in a circle of radius 4 m.

## Solution

$$\theta = \frac{s}{r} = \frac{6}{4} = \frac{3}{2} \text{ rad}$$



## Problems

**- Find the radian measure of the angle with the given degree measure.**

**(a)** $15°$

$$15° = 15 \times \frac{\pi}{180}$$

$$= \frac{\pi}{12} \text{ rad}$$

**(b)** $-45°$

$$-45° = -45 \times \frac{\pi}{180}$$

$$= -\frac{\pi}{4} \text{ rad}$$

**(c)** $36°$

$$36° = 36 \times \frac{\pi}{180}$$

$$= \frac{\pi}{5} \text{ rad}$$

**- Find the degree measure of the angle with the given radian measure.**

**(a)** $\frac{5\pi}{3}$

$$\frac{5\cancel{\pi}}{\cancel{3}} \times \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}} = 300^\circ$$

**(b)** $\frac{3\pi}{4}$

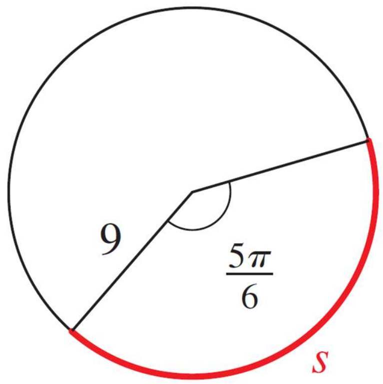
$$\frac{3\cancel{\pi}}{\cancel{4}} \times \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}} = 135^\circ$$

**(c)** $\frac{5\pi}{6}$

$$\frac{5\cancel{\pi}}{\cancel{6}} \times \frac{\overset{30}{\cancel{180}}}{\cancel{\pi}} = 150^\circ$$

**- Find the length $s$ of the circular arc, the radius $r$ of the circle, or the central angle $\theta$, as indicated.**

**(a)**

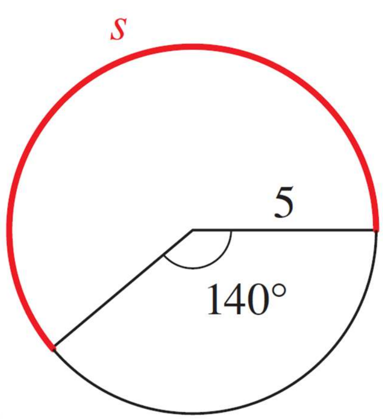



$s = r\theta$

$= 9 \; \frac{5\pi}{6}$

$= \frac{15\pi}{2}$ m

**(b)**




$\theta = 360^\circ - 140^\circ = 220^\circ$

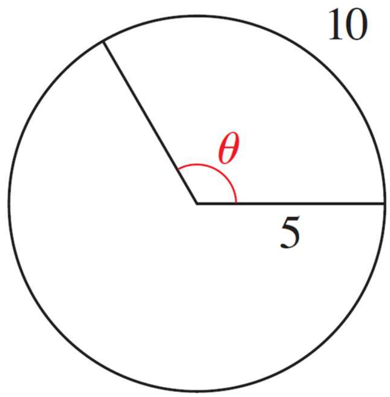
$220^\circ \times \frac{\pi}{180^\circ} = \frac{11\pi}{9}$

$s = r\theta$

$= 5 \cdot \frac{11\pi}{9}$

$= \frac{55\pi}{9}$ m

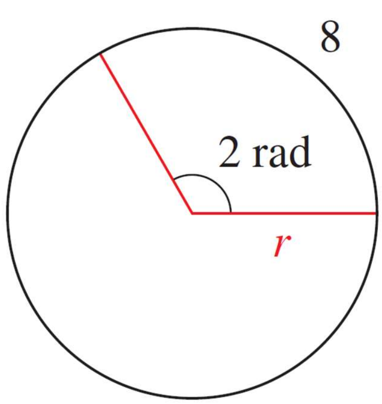
**(c)**




$\theta = \frac{s}{r}$

$= \frac{10}{5} = 2$ rad

**(d)**




$r = \frac{s}{\theta}$

$= \frac{8}{2} = 4$ m

**- A central angle $\theta$ in a circle of radius 9 m is subtended by an arc of length 14 m. Find the measure of $\theta$ in degrees and radians.**

$$\theta = \frac{s}{r}$$

$$= \frac{14}{9} \text{ rad}$$

$$\theta = \frac{14}{9} \cdot \frac{180}{\pi} = \frac{280}{\pi}^{\circ}$$