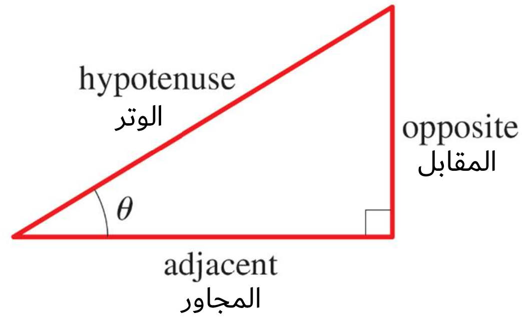
Section 6.2 – Trigonometry of Right Triangles

- في المثلث قائم الزاوية right triangle تكون الدوال المثلثية trigonometric ratios للزاوية θ كالتالي:

$$\sin\theta = \frac{Opposite}{Hypotenuse} = \frac{\text{مقابل}}{\text{وتر}}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse} = \frac{\text{مجاور}}{\text{وتر}}$$

$$\tan\theta = \frac{Opposite}{Adjacent} = \frac{\text{مقابل}}{\text{مجاور}} = \frac{\sin\theta}{\cos\theta}$$



$$\csc\theta = \frac{Hypotenuse}{Opposite} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{Hypotenuse}{Adjacent} = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{Adjacent}{Opposite} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

تذكر

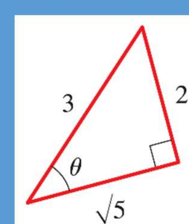
$$adj = \sqrt{hyp^2 - opp^2}$$

$$opp = \sqrt{hyp^2 - adj^2}$$

$$hyp = \sqrt{opp^2 + adj^2}$$

Example 1

Find the six trigonometric ratios of the angle θ in the figure.



Solution

$\sin\theta = \frac{2}{3}$

$\cos\theta = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{2}{\sqrt{5}}$

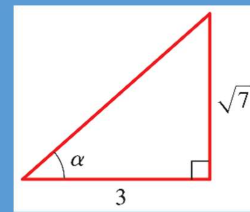
$\csc\theta = \frac{1}{\sin\theta} = \frac{3}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{3}{\sqrt{5}}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{5}}{2}$

Example 2

Find the six trigonometric ratios of the angle α in the figure.



Solution

$$\text{hyp.} = \sqrt{(\sqrt{7})^2 + 3^2} = \sqrt{7+9} = \sqrt{16} = 4$$

$$\sin\alpha = \frac{\sqrt{7}}{4} \qquad \csc\alpha = \frac{4}{\sqrt{7}}$$

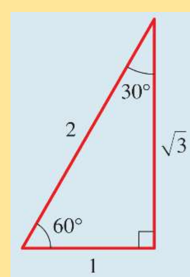
$$\cos\alpha = \frac{3}{4} \qquad \sec\alpha = \frac{4}{3}$$

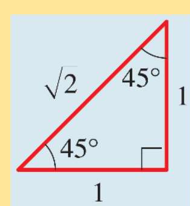
$$\tan\alpha = \frac{\sqrt{7}}{3} \qquad \cot\alpha = \frac{3}{\sqrt{7}}$$

لاحظ

توجد حالتين خاصتين من المثلثات القائمة **Special Triangles**

θ (Degreees)	θ (Radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	–





Example 3

Find the exact value of the following.

(a) $\sin\frac{\pi}{6} + \cos\frac{\pi}{6}$

(b) $(\cos 30°)^2 - (\sin 30°)^2$

Solution

(a) $\sin\frac{\pi}{6} + \cos\frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

(b) $\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

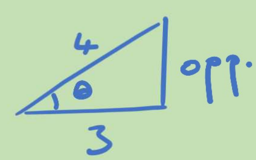
Example 4

Given $\cos\theta = \frac{3}{4}$ and θ is an acute angle. Find all the other trigonometric ratios.

Solution

$\text{opp.} = \sqrt{4^2 - 3^2}$

$= \sqrt{16-9} = \sqrt{7}$



$\sin\theta = \frac{\sqrt{7}}{4}$

$\csc\theta = \frac{4}{\sqrt{7}}$

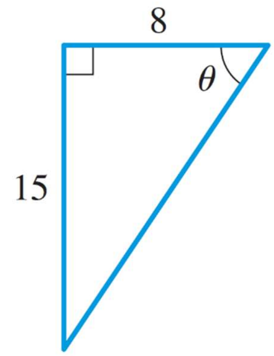
$\sec\theta = \frac{4}{3}$

$\tan\theta = \frac{\sqrt{7}}{3}$

$\cot\theta = \frac{3}{\sqrt{7}}$

Problems

- Find the exact values of the six trigonometric ratios of the angle θ in the triangle.



$$\text{hyp.} = \sqrt{15^2 + 8^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289} = 17$$

$\sin\theta = \dfrac{15}{17}$ $\qquad$ $\csc\theta = \dfrac{17}{15}$

$\cos\theta = \dfrac{8}{17}$ $\qquad$ $\sec\theta = \dfrac{17}{8}$

$\tan\theta = \dfrac{15}{8}$ $\qquad$ $\cot\theta = \dfrac{8}{15}$

- **Evaluate the expression.**

(a) $\sin 30° \csc 30°$

$\cancel{\sin 30} \cdot \frac{1}{\cancel{\sin 30}} = 1$

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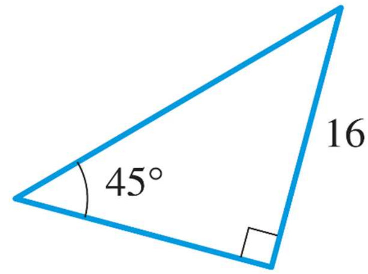
$\frac{1}{2} \cdot 2 = 1$

(b) $\sin 30° \cos 60° + \sin 60° \cos 30°$

$\frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

- **Solve the right triangle.**



$\therefore \theta = 45°$

$\therefore adj = opp = 16$

$hyp = \sqrt{16^2 + 16^2} = 16\sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}}$ $\qquad$ $\csc\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\qquad$ $\sec\theta = \sqrt{2}$

$\tan\theta = 1$ $\qquad$ $\cot\theta = 1$