

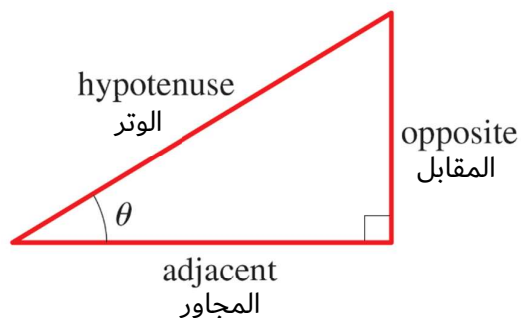
Section 6.2 – Trigonometry of Right Triangles

- في المثلث قائم الزاوية right triangle تكون الدوال المثلثية trigonometric ratios للزاوية θ كالتالي:

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{\text{مقابل}}{\text{وتر}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\text{مجاور}}{\text{وتر}}$$

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\text{مقابل}}{\text{مجاور}} = \frac{\sin \theta}{\cos \theta}$$



تذكر

$$\csc \theta = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

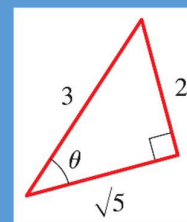
$$\text{adj} = \sqrt{\text{hyp}^2 - \text{opp}^2}$$

$$\text{opp} = \sqrt{\text{hyp}^2 - \text{adj}^2}$$

$$\text{hyp} = \sqrt{\text{opp}^2 + \text{adj}^2}$$

Example 1

Find the six trigonometric ratios of the angle θ in the figure.



Solution

$$\sin \theta = \frac{2}{3}$$

$$\cos \theta = \frac{\sqrt{5}}{3}$$

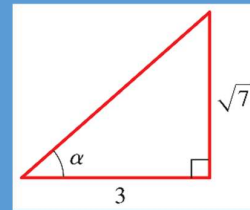
$$\tan \theta = \frac{2}{\sqrt{5}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{3}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{3}{\sqrt{5}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\sqrt{5}}{2}$$

Example 2

Find the six trigonometric ratios of the angle α in the figure.

Solution

$$\text{hyp.} = \sqrt{(\sqrt{7})^2 + 3^2} = \sqrt{7 + 9} = \sqrt{16} = 4$$

$$\sin \alpha = \frac{\sqrt{7}}{4}$$

$$\csc \alpha = \frac{4}{\sqrt{7}}$$

$$\cos \alpha = \frac{3}{4}$$

$$\sec \alpha = \frac{4}{3}$$

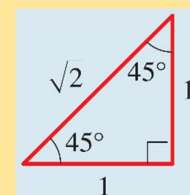
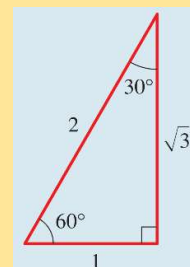
$$\tan \alpha = \frac{\sqrt{7}}{3}$$

$$\cot \alpha = \frac{3}{\sqrt{7}}$$

لاحظ

توجد حالتين خاصتين من المثلثات القائمة **Special Triangles**

θ (Degrees)	θ (Radians)	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	—



Example 3

Find the exact value of the following.

(a) $\sin \frac{\pi}{6} + \cos \frac{\pi}{6}$

(b) $(\cos 30^\circ)^2 - (\sin 30^\circ)^2$

Solution

$$(a) \quad \sin \frac{\pi}{6} + \cos \frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

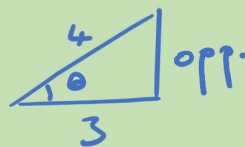
$$(b) \quad \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Example 4

Given $\cos \theta = \frac{3}{4}$ and θ is an acute angle. Find all the other trigonometric ratios.

Solution

$$\begin{aligned} \text{opp.} &= \sqrt{4^2 - 3^2} \\ &= \sqrt{16 - 9} = \sqrt{7} \end{aligned}$$



$$\sin \theta = \frac{\sqrt{7}}{4}$$

$$\csc \theta = \frac{4}{\sqrt{7}}$$

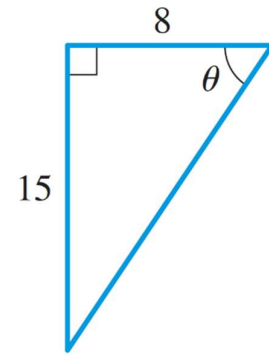
$$\sec \theta = \frac{4}{3}$$

$$\tan \theta = \frac{\sqrt{7}}{3}$$

$$\cot \theta = \frac{3}{\sqrt{7}}$$

Problems

- Find the exact values of the six trigonometric ratios of the angle θ in the triangle.



- Evaluate the expression.

(a) $\sin 30^\circ \csc 30^\circ$

(b) $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ$

- Solve the right triangle.

