

Section 6.3 – Trigonometric Functions of Angles

- بشكل عام نستطيع حساب الدوال المثلثية trigonometric ratios لأي زاوية θ كالآتي:

$$\sin \theta = \frac{y}{r}$$

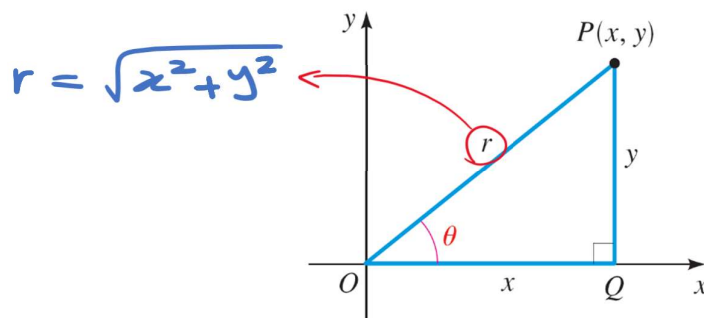
$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

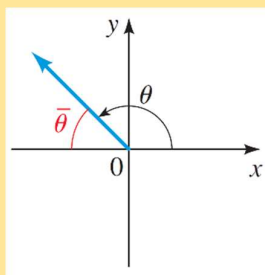
$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$



لاحظ

$\bar{\theta}$ تسمى reference angle و تتحدد قيمتها حسب موضع الزاوية، موضع الزاوية يحدد إشارة الدالة المثلثية



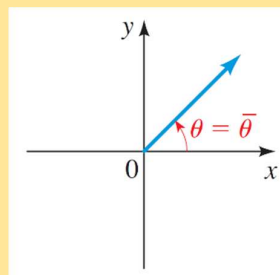
Quadrant II

الربع الثاني

$$\frac{\pi}{2} < \theta < \pi$$

$$\bar{\theta} = \pi - \theta$$

موجب $\sin$



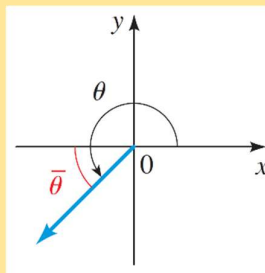
Quadrant I

الربع الأول

$$0 < \theta < \frac{\pi}{2}$$

$$\bar{\theta} = \theta$$

الكل موجب



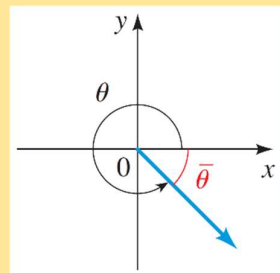
Quadrant III

الربع الثالث

$$\pi < \theta < \frac{3\pi}{2}$$

$$\bar{\theta} = \theta - \pi$$

موجب $\tan$



Quadrant IV

الربع الرابع

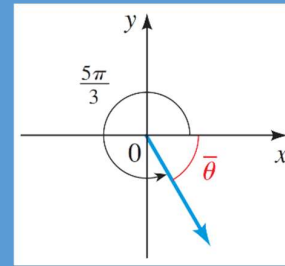
$$\frac{3\pi}{2} < \theta < 2\pi$$

$$\bar{\theta} = 2\pi - \theta$$

موجب $\cos$

Example 1

Find the reference angle for (a) $\theta = \frac{5\pi}{3}$ and (b) $\theta = 200^\circ$.



Solution

$$(a) \bar{\theta} = 2\pi - \frac{5\pi}{3} = \frac{3 \cdot 2\pi - 5\pi}{3} = \frac{\pi}{3}$$

$$(b) \bar{\theta} = 200^\circ - 180^\circ = 20^\circ$$

- لإيجاد الدوال المثلثية trigonometric functions لأي زاوية θ تتبع الخطوات التالية:

- 1- أوجد الزاوية $\bar{\theta}$ (reference angle).
- 2- حدد إشارة الدالة بناء على موضع الزاوية (أي ربع).
- 3- قيمة الدالة المثلثية للزاوية هي نفس قيمتها ل $\bar{\theta}$ مع أخذ الإشارة من الخطوة السابقة.

Example 2

Find (a) $\cos 135^\circ$ (b) $\sin \frac{4\pi}{3}$

Solution

$$(a) \bar{\theta} = 180^\circ - 135^\circ = 45^\circ$$

$\cos 135^\circ$ is negative (Quadrant II)

$$\cos 135^\circ = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

$$(b) \bar{\theta} = \frac{4\pi}{3} - \pi = \frac{4\pi - 3\pi}{3} = \frac{\pi}{3}$$

$\sin \frac{4\pi}{3}$ is negative (Quadrant III)

$$\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

- Trigonometric identities الاقترانات المثلثية

Reciprocal Identities

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

Example 3

Given $\sin \theta = -\frac{4}{5}$ and θ is in quadrant III, find: $\csc \theta$, $\cos \theta$, $\cot \theta$.

Solution

$$\csc \theta = -\frac{5}{4}$$

$$\cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$= -\sqrt{1 - \left(-\frac{4}{5}\right)^2}$$

$$= -\sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}}$$

$$= -\frac{3}{5}$$

negative
because θ is
in quadrant III

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = -\frac{3}{5} \div -\frac{4}{5} = -\frac{3}{5} \cdot -\frac{5}{4} = \frac{3}{4}$$

Problems

- Find the exact values of the six trigonometric function.

(a) $\cos 150^\circ$

(b) $\tan 330^\circ$

(c) $\sin \frac{3\pi}{2}$

(d) $\tan \left(-\frac{4\pi}{3} \right)$

- Write the first trigonometric function in terms of the second for θ in the given quadrant.

(a) $\tan \theta$, $\cos \theta$; θ in Quadrant III

(b) $\cot \theta$, $\sin \theta$; θ in Quadrant II

- Given $\cos \theta = -\frac{2}{7}$ and $\tan \theta < 0$, find: $\sin \theta$, $\sec \theta$, $\tan \theta$

- Given $\sec \theta = 2$ and θ in Quadrant IV, find: $\csc \theta$, $\cos \theta$, $\tan \theta$