

Section 7.1 – Trigonometric Identities

- Trigonometric identities المثلثية are equations relating trigonometric functions of angles as follows:

Reciprocal identities

$$\begin{aligned} \csc x &= \frac{1}{\sin x} & \sec x &= \frac{1}{\cos x} & \cot x &= \frac{1}{\tan x} \\ \tan x &= \frac{\sin}{\cos x} & \cot x &= \frac{\cos x}{\sin} \end{aligned}$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

Even-Odd identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

Cofunction identities

$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x & \sec\left(\frac{\pi}{2} - x\right) &= \csc x \\ \cos\left(\frac{\pi}{2} - x\right) &= \sin x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x & \csc\left(\frac{\pi}{2} - x\right) &= \sec x \end{aligned}$$

Example 1

Simplify the expression $\cos t + \tan t \sin t$.

Solution

$$\begin{aligned} &\cos t + \frac{\sin t}{\cos t} \cdot \sin t \\ &= \cos t + \frac{\sin^2 t}{\cos t} = \frac{\cos^2 t}{\cos t} + \frac{\sin^2 t}{\cos t} \\ &= \frac{\cos^2 t + \sin^2 t}{\cos t} = \frac{1}{\cos t} = \sec t \end{aligned}$$

Example 2

Simplify the expression.

(a) $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$
 (b) $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$

Solution

$$(a) \quad \frac{(\sin \theta)(1 + \sin \theta) + \cos \theta \cos \theta}{(\cos \theta)(1 + \sin \theta)}$$

$$= \frac{\sin \theta + \sin^2 \theta + \cos^2 \theta}{(\cos \theta)(1 + \sin \theta)}$$

$$= \frac{\cancel{\sin \theta} + 1}{(\cos \theta)(\cancel{1 + \sin \theta})} = \frac{1}{\cos \theta} = \sec \theta$$

$$(b) \quad \frac{(1 + \sin x)(1 + \sin x) + \cos x \cos x}{(\cos x)(1 + \sin x)}$$

$$= \frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{(\cos x)(1 + \sin x)}$$

$$= \frac{1 + 2\sin x + 1}{(\cos x)(1 + \sin x)} = \frac{2 + 2\sin x}{(\cos x)(1 + \sin x)}$$

$$= \frac{2(\cancel{1 + \sin x})}{(\cos x)(\cancel{1 + \sin x})} = \frac{2}{\cos x} = 2 \sec x$$

Example 3

Verify (prove) the identity.

(a) $\frac{\cos \theta}{\sec \theta \sin \theta} = \csc \theta - \sin \theta$

(b) $\cos(-t) - \sin(-t) = \cos t + \sin t$

Solution

$$(a) \text{ R H S } = \csc \theta - \sin \theta$$

$$= \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta}$$

$$\text{L H S } = \frac{\cos \theta}{\frac{1}{\cos \theta} \sin \theta} = \cos \theta \div \frac{\sin \theta}{\cos \theta}$$

$$= \cos \theta \cdot \frac{\cos \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta} = \text{R H S}$$

$$(b) \cos(-t) = \cos t$$

$$\sin(-t) = -\sin t$$

$$\therefore \cos(-t) - \sin(-t)$$

$$= \cos t - (-\sin t)$$

$$= \cos t + \sin t = \text{R H S}$$

Problems

- Simplify the expression.

(a) $\tan^2 x - \sec^2 x$

(b) $\frac{\sin x \sec x}{\tan x}$

- Verify (prove) the identity.

(a) $\cos \theta (\sec \theta \cos \theta) = \sin^2 \theta$

(b) $\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$

$$(c) (\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$$

$$(d) \frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta} = 1$$

$$(e) \frac{\cot \theta}{\csc \theta - \sin \theta} = \sec \theta$$

$$(f) \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$