

Section 7.2 – Addition and Subtraction Formulas

- Trigonometric functions الدوال المثلثية of sums and differences are as follows:

Formulas for sine

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s - t) = \sin s \cos t - \cos s \sin t$$

Formulas for cosine

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

Example 1

Find the exact value of the expression.

(a) $\cos 75^\circ$

(b) $\cos \frac{\pi}{12}$

Solution

$$(a) \cos 75^\circ = \cos(45^\circ + 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{2}\sqrt{3}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$(b) \cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Example 2

Find the exact value of the expression.

(a) $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$

(b) $\cos\left(\frac{3\pi}{7}\right) \cos\left(\frac{2\pi}{21}\right) + \sin\left(\frac{3\pi}{7}\right) \sin\left(\frac{2\pi}{21}\right)$

Solution

$$\begin{aligned} \text{(a)} \quad & \sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ \\ &= \sin (20^\circ + 40^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \cos\left(\frac{3\pi}{7}\right) \cos\left(\frac{2\pi}{21}\right) + \sin\left(\frac{3\pi}{7}\right) \sin\left(\frac{2\pi}{21}\right) \\ &= \cos\left(\frac{3\pi}{7} - \frac{2\pi}{21}\right) \\ &= \cos\left(\frac{3 \cdot 3\pi - 2\pi}{21}\right) \\ &= \cos\left(\frac{9\pi - 2\pi}{21}\right) \\ &= \cos\left(\frac{7\pi}{21}\right) \\ &= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \end{aligned}$$

Example 3

Verify (prove) the idea.

(a) $\cos\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{6}\right) = 0$

(b) $\sin\left(\theta - \frac{\pi}{2}\right) = -\cos \theta$

Solution

$$(a) \cos\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{6}\right)$$

$$= \cos x \cos\left(\frac{\pi}{3}\right) - \sin x \sin \frac{\pi}{3} + \sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}$$

$$= \cancel{\frac{1}{2} \cos x} - \cancel{\frac{\sqrt{3}}{2} \sin x} + \cancel{\frac{\sqrt{3}}{2} \sin x} - \cancel{\frac{1}{2} \cos x}$$

$$= 0 = R.H.S$$

$$(b) \sin\left(\theta - \frac{\pi}{2}\right) = \sin \theta \overset{0}{\cancel{\cos \frac{\pi}{2}}} - \cos \theta \sin \frac{\pi}{2}$$

$$= -1 \cdot \cos \theta$$

$$= -\cos \theta = R.H.S$$

Problems

- Verify (prove) the identity.

(a) $\sin(x + y) - \sin(x - y) = 2 \cos x \sin y$

(b) $\cos(x - \pi) = -\cos x$