

Section 7.3 – Double-Angle Formulas

- نستخدم **Double-Angle Formulas** لإيجاد الدوال المثلثية لضعف قيمة زاوية $2x$ من الزاوية نفسها x

Formulas for sine

$$\sin 2x = 2 \sin x \cos x$$

Formulas for cosine

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2 \sin^2 x$$

$$= 2 \cos^2 x - 1$$

Example 1

Simplify.

(a) $2 \sin(3x) \cos(3x)$

(b) $\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$

Solution

$$\begin{aligned} \text{(a)} \quad 2 \sin(3x) \cos(3x) &= \sin(2 \cdot 3x) \\ &= \sin(6x) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) \\ = \cos\left(2 \cdot \frac{\theta}{2}\right) = \cos \theta \end{aligned}$$

Example 2

If $\cos x = -\frac{2}{3}$ and x is in Quadrant II, find $\cos 2x$ and $\sin 2x$

Solution

$$\cos x = -\frac{2}{3} = -\frac{x}{r}$$

$$y = \sqrt{r^2 - x^2}$$

$$= \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$$

$$\sin x = \frac{\sqrt{5}}{3} \quad \text{positive in quadrant II}$$

$$\cos 2x = 2\cos^2 x - 1$$

$$= 2\left(-\frac{2}{3}\right)^2 - 1$$

$$= 2\left(\frac{4}{9}\right) - 1$$

$$= \frac{8}{9} - 1 = \frac{8}{9} - \frac{9}{9} = -\frac{1}{9}$$

$$\sin 2x = 2\sin x \cos x$$

$$= 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$$

$$= -\frac{4\sqrt{5}}{9}$$

Problems

- Find $\cos 2x$ and $\sin 2x$ from the given information.

$$\sin x = \frac{5}{13}, x \text{ is in Quadrant I.}$$