Section 12.4 – Mathematics of Finance

- Amount of an Annuity الاستحقاق السنوي:

المجموع (المبلغ النهائي)

عدد الدفعات

$$A_f = R\frac{(1+i)^n - 1}{i}$$

قيمة القسط

الفايدة

Example 1

How much money should be invested every month at 12% per year, compounded monthly, in order to have $4000 in 18 months?

Solution

$$i = \frac{0.12}{12} = 0.01$$

$$A_f = R\frac{(1+i)^n - 1}{i}$$

$$4000 = R\frac{(1+0.01)^{18} - 1}{0.01}$$

$$R = \frac{0.01 \times 4000}{(1.01)^{18} - 1}$$

$$R = \frac{40}{(1.01)^{18} - 1} \approx \$203.93$$

- The present value of an annuity:

$$A_p = R\frac{1-(1+i)^{-n}}{i}$$

القيمة الحالية

Example 2

A person wins \$10,000,000 in the California lottery, and the amount is paid in yearly installments of half a million dollars each for 20 years. What is the present value of his winnings? Assume that he can earn 10% interest, compounded annually.

Solution

$$A_p = 500{,}000\,\frac{1-(1+0.1)^{-20}}{0.1}$$

$$= 500{,}000\,\frac{1-\frac{1}{(1.1)^{20}}}{0.1} \approx \$4{,}256{,}782$$

Example 3

A student wishes to buy a car. She can afford to pay \$200 per month but has no money for a down payment. If she can make these payments for 4 years and the interest rate is 12%, what purchase price can she afford?

Solution

$$i = \frac{0.12}{12} = 0.01, \quad n = 4 \times 12 = 48$$

$$A_p = 200\,\frac{1-(1+0.01)^{-48}}{0.01}$$

$$= 200\,\frac{1-\frac{1}{(1.01)^{48}}}{0.01} \approx \$7594.79$$

- Installment buying حساب الأقساط:

$$R = \frac{iA_p}{1-(1+i)^{-n}}$$

Example 4

A couple borrows \$100,000 at 9% interest as a mortgage loan on a house. They expect to make monthly payments for 30 years to repay the loan. What is the size of each payment?

Solution

$i = \frac{0.09}{12} = 0.0075$, $n = 30 \times 12 = 360$

$$R = \frac{iA_p}{1-(1+i)^{-n}}$$

$$= \frac{0.0075 \times 100{,}000}{1-(1+0.0075)^{-360}} = \$804.62$$

- لحساب القيمة المستقبلية في حالة عدم وجود أقساط (R)

$$A_p = A_f(1+i)^{-n}$$

Example 5

How much money must be invested now at 12% per year, compounded monthly, to receive \$5000 due in two years?

Solution

$i = \frac{0.12}{12} = 0.01$, $n = 12 \times 2 = 24$

$$A_p = 5000(1+0.01)^{-24}$$

$$= 5000(1.01)^{-24} \approx \$3{,}937.83$$

Problems

- Find the amount of an annuity that consists of ten annual payments of $1000 each into an account that pays 6% interest per year.

$$R = 1000, \quad n = 10, \quad i = 0.06$$

$$A_f = 1000 \, \frac{(1+0.06)^{10} - 1}{0.06}$$

$$\approx \$\,13{,}180.79$$

- How much money should be invested every quarter at 10% per year, compounded quarterly, to have $5000 in 2 years?

$$A_f = 5000, \quad i = \frac{0.1}{12} = 0.025, \quad n = 2 \times 4 = 8$$

$$5000 = R \, \frac{(1+0.025)^8 - 1}{0.025}$$

$$R = \frac{0.025 \times 5000}{(1+0.025)^8 - 1}$$

$$\approx \$\,572.34$$

- What is the present value of an annuity that consists of 20 semiannual payments of $\$1000$ at an interest rate of 9% per year, compounded semiannually?

$$R = 1000, \quad i = \frac{0.09}{2} = 0.045, \quad n = 20$$

$$A_p = 1000 \, \frac{1-(1+0.045)^{-20}}{0.045}$$

$$\approx \$13{,}007.94$$

- A woman wants to borrow $\$12,000$ to buy a car. She wants to repay the loan by monthly installments for 4 years. If the interest rate on this loan is $10\frac{1}{2}\%$ per year, compounded monthly, what is the amount of each payment?

$$A_p = 12{,}000, \quad n = 4 \times 12 = 48, \quad i = \frac{0.105}{12} = 0.00875$$

$$12000 = R \, \frac{1-(1+0.00875)^{-48}}{0.00875}$$

$$R = \frac{0.00875 \times 12000}{1-(1+0.00875)^{-48}}$$

$$\approx \$307.24$$

- Dr. Gupta is considering a 30-year mortgage at 6% interest. She can make payments of $\$3500$ a month. What size loan can she afford?

$n = 30 \times 12 = 360$, $i = \frac{0.06}{12} = 0.005$, $R = 3500$

$$A_p = 3500 \, \frac{1 - (1 + 0.005)^{-360}}{0.005}$$

$$\approx 583{,}770.65$$