

Section 12.4 – Mathematics of Finance

- Amount of an Annuity السنوي الاستحقاق:

$$A_f = R \frac{(1+i)^n - 1}{i}$$

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قيمة القسط الفائدة

Example 1

How much money should be invested every month at 12% per year, compounded monthly, in order to have \$4000 in 18 months?

Solution

$$i = \frac{0.12}{12} = 0.01$$

$$A_f = R \frac{(1+i)^n - 1}{i}$$

$$4000 = R \frac{(1+0.01)^{18} - 1}{0.01}$$

$$R = \frac{0.01 \times 4000}{(1.01)^{18} - 1}$$

$$R = \frac{40}{(1.01)^{18} - 1} \approx \$203.93$$

- The present value of an annuity:

$$A_p = R \frac{1 - (1 + i)^{-n}}{i}$$

القيمة الحالية

Example 2

A person wins \$10,000,000 in the California lottery, and the amount is paid in yearly installments of half a million dollars each for 20 years. What is the present value of his winnings? Assume that he can earn 10% interest, compounded annually.

Solution

$$A_p = 500,000 \frac{1 - (1 + 0.1)^{-20}}{0.1}$$

$$= 500,000 \frac{1 - \frac{1}{(1.1)^{20}}}{0.1} \approx \$4,256,782$$

Example 3

A student wishes to buy a car. She can afford to pay \$200 per month but has no money for a down payment. If she can make these payments for 4 years and the interest rate is 12%, what purchase price can she afford?

Solution

$$i = \frac{0.12}{12} = 0.01, n = 4 \times 12 = 48$$

$$A_p = 200 \frac{1 - (1 + 0.01)^{-48}}{0.01}$$

$$= 200 \frac{1 - \frac{1}{(1.01)^{48}}}{0.01} \approx \$7594.79$$

- Installment buying حساب الأقساط:

$$R = \frac{iA_p}{1 - (1 + i)^{-n}}$$

Example 4

A couple borrows \$100,000 at 9% interest as a mortgage loan on a house. They expect to make monthly payments for 30 years to repay the loan. What is the size of each payment?

Solution

$$i = \frac{0.09}{12} = 0.0075, \quad n = 30 \times 12 = 360$$

$$\begin{aligned} R &= \frac{iA_p}{1 - (1 + i)^{-n}} \\ &= \frac{0.0075 \times 100,000}{1 - (1 + 0.0075)^{-360}} = \$ 804.62 \end{aligned}$$

- لحساب القيمة المستقبلية في حالة عدم وجود أقساط (R)

$$A_p = A_f(1 + i)^{-n}$$

Example 5

How much money must be invested now at 12% per year, compounded monthly, to receive \$5000 due in two years?

Solution

$$i = \frac{0.12}{12} = 0.01, \quad n = 12 \times 2 = 24$$

$$\begin{aligned} A_p &= 5000 (1 + 0.01)^{-24} \\ &= 5000 (1.01)^{-24} = \$ 3,937.83 \end{aligned}$$

Problems

- Find the amount of an annuity that consists of ten annual payments of \$1000 each into an account that pays 6% interest per year.

- How much money should be invested every quarter at 10% per year, compounded quarterly, to have \$5000 in 2 years?

- What is the present value of an annuity that consists of 20 semiannual payments of \$1000 at an interest rate of 9% per year, compounded semiannually?

- A woman wants to borrow \$12,000 to buy a car. She wants to repay the loan by monthly installments for 4 years. If the interest rate on this loan is $10\frac{1}{2}\%$ per year, compounded monthly, what is the amount of each payment?

- Dr. Gupta is considering a 30-year mortgage at 6% interest. She can make payments of \$3500 a month. What size loan can she afford?