Section 6.3 – Trigonometric Functions of Angles

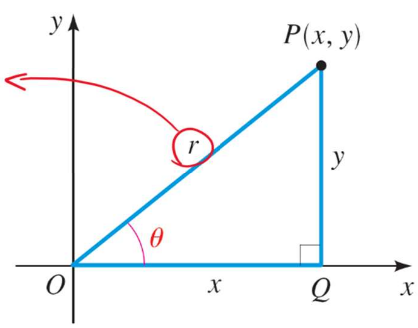
- بشكل عام نستطيع حساب الدوال المثلثية trigonometric ratios لأي زاوية θ كالتالي:

$\sin\theta = \dfrac{y}{r}$

$\cos\theta = \dfrac{x}{r}$

$\tan\theta = \dfrac{y}{x}$

$r = \sqrt{x^2+y^2}$



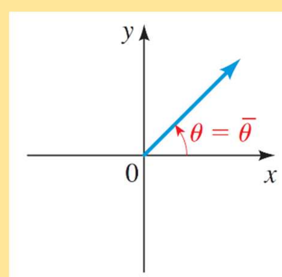
$\csc\theta = \dfrac{r}{y}$

$\sec\theta = \dfrac{r}{x}$

$\cot\theta = \dfrac{x}{y}$

لاحظ

$\bar{\theta}$ تسمى <u>reference angle</u> و تتحدد قيمتها حسب موضع الزاوية، موضع الزاوية يحدد إشارة الدالة المثلثية



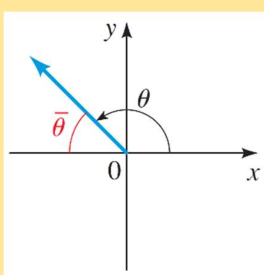
Quadrant I

الربع الأول

$0 < \theta < \dfrac{\pi}{2}$

$\bar{\theta} = \theta$

الكل موجب



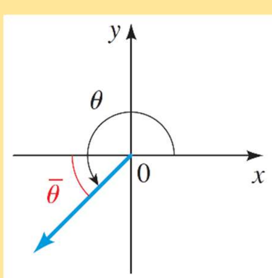
Quadrant II

الربع الثاني

$\dfrac{\pi}{2} < \theta < \pi$

$\bar{\theta} = \pi - \theta$

موجب sin



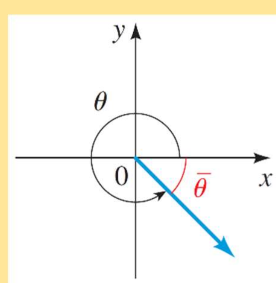
Quadrant III

الربع الثالث

$\pi < \theta < \dfrac{3\pi}{2}$

$\bar{\theta} = \theta - \pi$

موجب tan



Quadrant IV

الربع الرابع

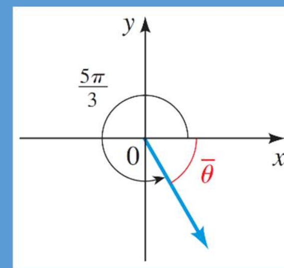
$\dfrac{3\pi}{2} < \theta < 2\pi$

$\bar{\theta} = 2\pi - \theta$

موجب cos

Example 1

Find the reference angle for (a) $\theta = \frac{5\pi}{3}$ and (b) $\theta = 200°$.



Solution

(a) $\bar{\theta} = 2\pi - \frac{5\pi}{3} = \frac{3.2\pi - 5\pi}{3} = \frac{\pi}{3}$

(b) $\bar{\theta} = 200° - 180° = 20°$

- لإيجاد الدوال المثلثية trigonometric functions لأي زاوية θ نتبع الخطوات التالية:

1- أوجد الزاوية $\bar{\theta}$ (reference angle).

2- حدد إشارة الدالة بناء على موضع الزاوية (أي ربع).

3- قيمة الدالة المثلثية للزاوية هي نفس قيمتها ل $\bar{\theta}$ مع أخذ الإشارة من الخطوة السابقة.

Example 2

Find (a) $\cos 135°$ (b) $\sin \frac{4\pi}{3}$

Solution

(a) $\bar{\theta} = 180° - 135° = 45°$

$\cos 135°$ is negative (Quadrant II)

$\cos 135 = -\cos 45° = -\frac{1}{\sqrt{2}}$

(b) $\bar{\theta} = \frac{4\pi}{3} - \pi = \frac{4\pi - 3.\pi}{3} = \frac{\pi}{3}$

$\sin \frac{4\pi}{3}$ is negative (Quadrant III)

$\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

- Trigonometric identities الاقترانات المثلثية

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

Example 3

Given $\sin\theta = -\frac{4}{5}$ and θ is in quadrant III, find: $\csc\theta$, $\cos\theta$, $\cot\theta$.

Solution

$$\csc\theta = -\frac{5}{4}$$

$$\cos\theta = -\sqrt{1 - \sin^2\theta}$$

negative because θ is in quadrant III

$$= -\sqrt{1 - \left(-\frac{4}{5}\right)^2}$$

$$= -\sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}}$$

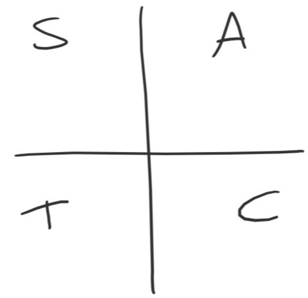
$$= -\frac{3}{5}$$

$$\cot = \frac{\cos\theta}{\sin\theta} = -\frac{3}{5} \div -\frac{4}{5} = -\frac{3}{\cancel{5}} \cdot -\frac{\cancel{5}}{4} = \frac{3}{4}$$

Problems

- Find the exact values of the trigonometric function.

(a) $\cos 150°$



$150°$ is in quadrant II

$\therefore \quad \bar{\theta} = 180° - 150° = 30°$

$\cos 150° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$

(b) $\tan 330°$

$330°$ is in quadrant IV

$\bar{\theta} = 360° - 330° = 30°$

$\tan 330° = -\tan 30°$

$= -\frac{1}{\sqrt{3}}$

(c) $\sin\frac{3\pi}{2}$

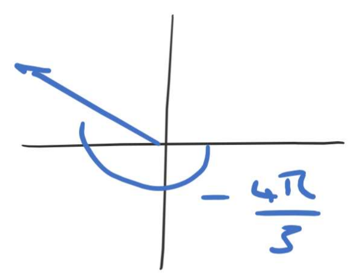
$\frac{3\pi}{2}$ is between quadrant III and IV

$\bar{\theta} = 2\pi - \frac{3\pi}{2} = \frac{3\pi}{2} - \pi = \frac{\pi}{2}$

$\sin\frac{3\pi}{2} = -\sin\frac{\pi}{2}$

$= -1$

(d) $\tan\left(-\frac{4\pi}{3}\right)$



$\theta = 2\pi - \frac{4\pi}{3} = \frac{2\pi}{3}$

$\bar{\theta} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\tan\left(-\frac{4\pi}{3}\right) = -\tan\frac{\pi}{3}$

$= -\sqrt{3}$

- Write the first trigonometric function in terms of the second for θ in the given quadrant.

(a) $\tan\theta$, $\cos\theta$; θ in Quadrant III

$$\tan\theta = \frac{-\sin\theta}{-\cos\theta}$$

$$= \frac{-\sqrt{1-\cos^2\theta}}{-\cos\theta}$$

(b) $\cot\theta$, $\sin\theta$; θ in Quadrant II

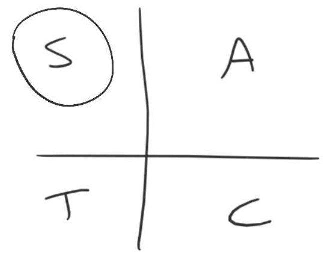
$$\cot\theta = \frac{-\cos\theta}{\sin\theta}$$

$$= \frac{-\sqrt{1-\sin^2\theta}}{\sin\theta}$$

- Given $\cos\theta = -\frac{2}{7}$ and $\tan\theta < 0$, find: $\sin\theta$, $\sec\theta$, $\tan\theta$

cos θ and tan θ both negative

∴ θ is in quadrant II



$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(-\frac{2}{7}\right)^2} = \sqrt{1-\frac{4}{49}}$$

$$= \sqrt{\frac{49}{49}-\frac{4}{49}} = \sqrt{\frac{45}{49}}$$

$$= \sqrt{\frac{9.5}{49}} = \frac{3\sqrt{5}}{7}$$

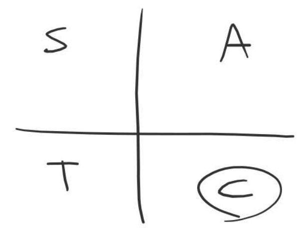
$$\sec\theta = \frac{1}{\cos\theta} = -\frac{7}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3\sqrt{5}}{7} \div -\frac{2}{7}$$

$$= \frac{3\sqrt{5}}{\cancel{7}} \cdot -\frac{\cancel{7}}{2} = -\frac{3\sqrt{5}}{2}$$

- Given $\sec\theta = 2$ and θ in Quadrant IV, find: $\csc\theta$, $\cos\theta$, $\tan\theta$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{2}$$



$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

$$= -\sqrt{1-\left(\frac{1}{2}\right)^2} = -\sqrt{1-\frac{1}{4}}$$

$$= -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{2}{\sqrt{3}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= -\frac{\sqrt{3}}{2} \div \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{\cancel{2}} \cdot \cancel{2} = -\sqrt{3}$$