Section 7.2 – Addition and Subtraction Formulas

- Trigonometric functions الدوال المثلثية of sums and differences are as follows:

Formulas for sine

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$

Formulas for cosine

$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$

Example 1

Find the exact value of the expression.

(a) $\cos 75°$

(b) $\cos \frac{\pi}{12}$

Solution

(a) $\cos 75° = \cos(45° + 30°)$

$= \cos 45 \cos 30 - \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{2}\sqrt{3}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$

(b) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4} \cdot \cos\frac{\pi}{6} + \sin\frac{\pi}{4} \cdot \sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

Example 2

Find the exact value of the expression.

(a) $\sin 20° \cos 40° + \cos 20° \sin 40°$

(b) $\cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{21}\right) + \sin\left(\frac{3\pi}{7}\right)\sin\left(\frac{2\pi}{21}\right)$

Solution

(a) $\sin 20° \cos 40° + \cos 20° \sin 40°$

$= \sin(20+40) = \sin 60° = \frac{\sqrt{3}}{2}$

(b) $\cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{21}\right) + \sin\left(\frac{3\pi}{7}\right)\sin\left(\frac{2\pi}{21}\right)$

$= \cos\left(\frac{3\pi}{7} - \frac{2\pi}{21}\right)$

$= \cos\left(\frac{3.3\pi - 2\pi}{21}\right)$

$= \cos\left(\frac{9\pi - 2\pi}{21}\right)$

$= \cos\left(\frac{7\pi}{21}\right)$

$= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Example 3

Verify (prove) the idea.

(a) $\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{6}\right)=0$

(b) $\sin\left(\theta-\frac{\pi}{2}\right)=-\cos\theta$

Solution

(a) $\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{6}\right)$

$= \cos x\cos\left(\frac{\pi}{3}\right)-\sin x\sin\frac{\pi}{3}+\sin x\cos\frac{\pi}{6}$
$-\cos x\sin\frac{\pi}{6}$

$= \cancel{\frac{1}{2}\cos x}-\cancel{\frac{\sqrt{3}}{2}\sin x}+\cancel{\frac{\sqrt{3}}{2}\sin x}-\cancel{\frac{1}{2}\cos x}$

$= 0 = RHS$

(b) $\sin\left(\theta-\frac{\pi}{2}\right)=\sin\theta\,\cancel{\cos\frac{\pi}{2}}^{\,0}-\cos\theta\sin\frac{\pi}{2}$

$= -1\cdot\cos\theta$

$= -\cos\theta = RHS$

Problems

- Verify (prove) the identity.

(a) $\sin(x+y) - \sin(x-y) = 2\cos x \sin y$

$$LHS = \sin(x+y) - \sin(x-y)$$

$$= \sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y)$$

$$= \cancel{\sin x \cos y} + \cos x \sin y - \cancel{\sin x \cos y} + \cos x \sin y$$

$$= 2\cos x \sin y = RHS$$

(b) $\cos(x-\pi) = -\cos x$

$$LHS = \cos(x-\pi)$$

$$= \cos x \underbrace{\cos \pi}_{-1} + \sin x \underbrace{\sin \pi}_{0}$$

$$= -\cos x = RHS$$