Section 7.3 – Double-Angle Formulas

- نستخدم Double-Angle Formulas لإيجاد الدوال المثلثية لضعف قيمة زاوية $2x$ من الزاوية نفسها x

Formulas for sine

$$\sin 2x = 2\sin x\cos x$$

Formulas for cosine

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

Example 1

Simplify.

(a) $2\sin(3x)\cos(3x)$

(b) $\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$

Solution

(a) $2\sin(3x)\cos(3x) = \sin(2\cdot 3x)$

$= \sin(6x)$

(b) $\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$

$= \cos\left(2\cdot\frac{\theta}{2}\right) = \cos\theta$

Example 2

If $\cos x = -\frac{2}{3}$ and x is in Quadrant II, find $\cos 2x$ and $\sin 2x$

Solution

$$\cos x = -\frac{2}{3} = -\frac{x}{r}$$

$$y = \sqrt{r^2 - x^2}$$

$$= \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$$

$$\sin x = \frac{\sqrt{5}}{3}$$ positive in quadrant II

$$\cos 2x = 2\cos^2 x - 1$$

$$= 2\left(-\frac{2}{3}\right)^2 - 1$$

$$= 2\left(\frac{4}{9}\right) - 1$$

$$= \frac{8}{9} - 1 = \frac{8}{9} - \frac{9}{9} = -\frac{1}{9}$$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$$

$$= -\frac{4\sqrt{5}}{9}$$

Problems

- Find $\cos 2x$ and $\sin 2x$ from the given information.

$\sin x = \frac{5}{13}$, x is in Quadrant I.

$$\sin x = \frac{y}{r}$$

$$x = \sqrt{r^2 - y^2} = \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25} = \sqrt{144} = 12$$

$$\cos x = \frac{x}{r} = \frac{12}{13}$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$= 1 - 2\left(\frac{5}{13}\right)^2$$

$$= 1 - 2 \cdot \frac{25}{169} = \frac{169}{169} - \frac{50}{169}$$

$$= \frac{119}{169}$$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$$

$$= \frac{120}{169}$$