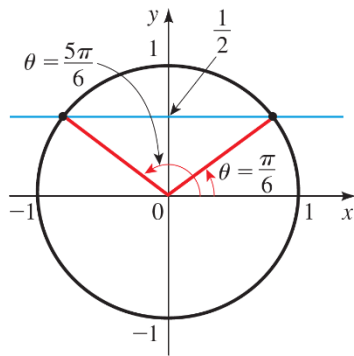


Section 7.4, 7.5 – Basic Trigonometric Equations, More Trigonometric Equations

- يوجد دائما أكثر من زاوية لهم نفس قيمة الدالة المثلثية trigonometric function

مثال:



$$\sin \frac{\pi}{6} = \sin \frac{5\pi}{6} = \frac{1}{2}$$

Example 1

- (a) Solve the equation $\sin \theta = \frac{1}{2}$
 (b) Solve the equation $\cos \theta = -\frac{\sqrt{2}}{2}$

Solution

(a) $\sin \theta$ is positive

therefore θ is in quadrant I and II

In quadrant I $\theta = \frac{\pi}{6}$

in quadrant II $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

(b) $\cos \theta$ is negative

$\therefore \theta$ in quadrant II and III

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\theta = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

لاحظ

في كل الأحوال تتكرر قيمة الدوال المثلثية $\sin \theta$ و $\cos \theta$ كل 2π ، وتتكرر قيمة $\tan \theta$ كل π

مثال:

$$\sin \frac{\pi}{2} = \sin \left(\frac{\pi}{2} + 2\pi \right) = \sin \frac{5\pi}{2}$$

$$\cos \frac{\pi}{2} = \cos \left(\frac{\pi}{2} + 2\pi \right) = \cos \frac{5\pi}{2}$$

$$\tan \frac{\pi}{2} = \tan \left(\frac{\pi}{2} + \pi \right) = \tan \frac{3\pi}{2}$$

Example 2

Find all solutions of the equation in the interval $[0, 2\pi)$

(a) $2 \sin \theta - 1 = 0$

(b) $\tan^2 \theta - 3 = 0$

Solution

(a) $2 \sin \theta = 1$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$$, \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

(b) $\tan^2 \theta = 3$

$$\tan \theta = \pm \sqrt{3}$$

$$\theta = \frac{\pi}{3}, \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{3} + \pi = \frac{4\pi}{3}, \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

Example 3

Solve the equation

(a) $2 \sin^2 \theta - \sin \theta - 1 = 0$

(b) $2 \cos 2\theta + 1 = 0$

Solution

(a) $(2 \sin \theta + 1)(\sin \theta - 1) = 0$

$2 \sin \theta + 1 = 0 \quad \text{or} \quad \sin \theta - 1 = 0$

$\sin \theta = -\frac{1}{2}$

$\sin \theta = 1$

$$\begin{aligned} \theta &= \frac{\pi}{6} + \pi \\ &= \frac{7\pi}{6} + 2k\pi \end{aligned}$$

$$\theta = \frac{\pi}{2} + 2k\pi$$

$$\begin{aligned} \theta &= 2\pi - \frac{\pi}{6} \\ &= \frac{11\pi}{6} + 2k\pi \end{aligned}$$

(b) $2 \cos 2\theta = -1$
 $\cos 2\theta = -\frac{1}{2}$

$$2\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{3} + 2k\pi$$

$$2\theta = \frac{\pi}{3} + \pi = \frac{4\pi}{3} \Rightarrow \theta = \frac{2\pi}{3} + 2k\pi$$

Problems

- Solve the equation

(a) $\sin \theta = \frac{\sqrt{3}}{2}$

(b) $\tan \theta = -\sqrt{3}$

(c) $\cos \theta = -\frac{\sqrt{3}}{2}$

- Find all solutions of the given equation.

(a) $\cos \theta + 1 = 0$

(b) $3 \tan^2 \theta - 1 = 0$

- Solve the given equation.

(a) $\cos^2 \theta - \cos \theta - 6 = 0$

- Use an Addition or Subtraction Formula to simplify the equation. Then find all solutions in the interval $[0, 2\pi)$.

(a) $\cos \theta \cos 3\theta + \sin \theta \sin 3\theta = 0$

(b) $2 \cos^2 \theta - 7 \cos \theta + 3 = 0$