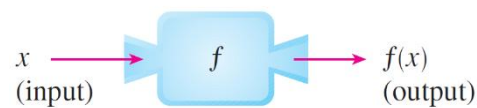
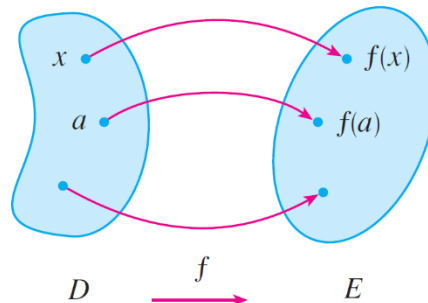


Section 1.2 – Mathematical Models: A Catalog of Essential Functions

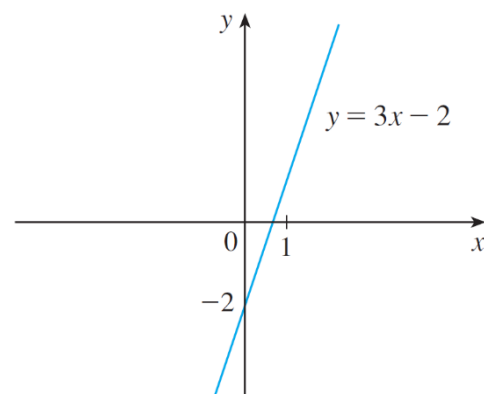
- الدوال Functions علاقة بين مجموعة مدخلات وتسمى المجال domain، ومجموعة مخرجات تسمى المدى range



- Linear functions الدوال الخطية (1/8):

$$y = f(x) = mx + b$$

x	$y = f(x) = 3x - 2$
1.0	1.0
1.1	1.3
1.2	1.6
1.3	1.9
1.4	2.2
1.5	2.5



- Polynomials الحدوديات، أو كثيرة الحدود (2/8):

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

Domain: $\mathbb{R} = (-\infty, \infty)$

Example

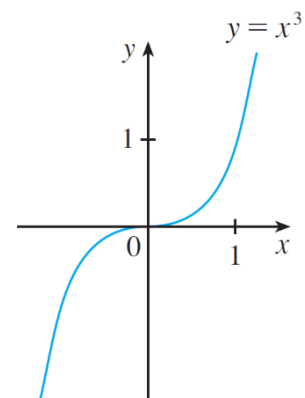
$$P(x) = 2x^6 - x^4 + \frac{2}{5}x^3 + \sqrt{2}$$

Degree = 6

Example

$$P(x) = ax^3 + bx^2 + cx + d, \quad a \neq 0$$

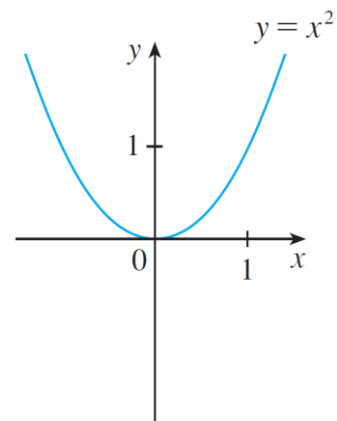
Degree = 3, cubic



Example

$$P(x) = ax^2 + bx + c, \quad a \neq 0$$

Degree = 2, quadratic



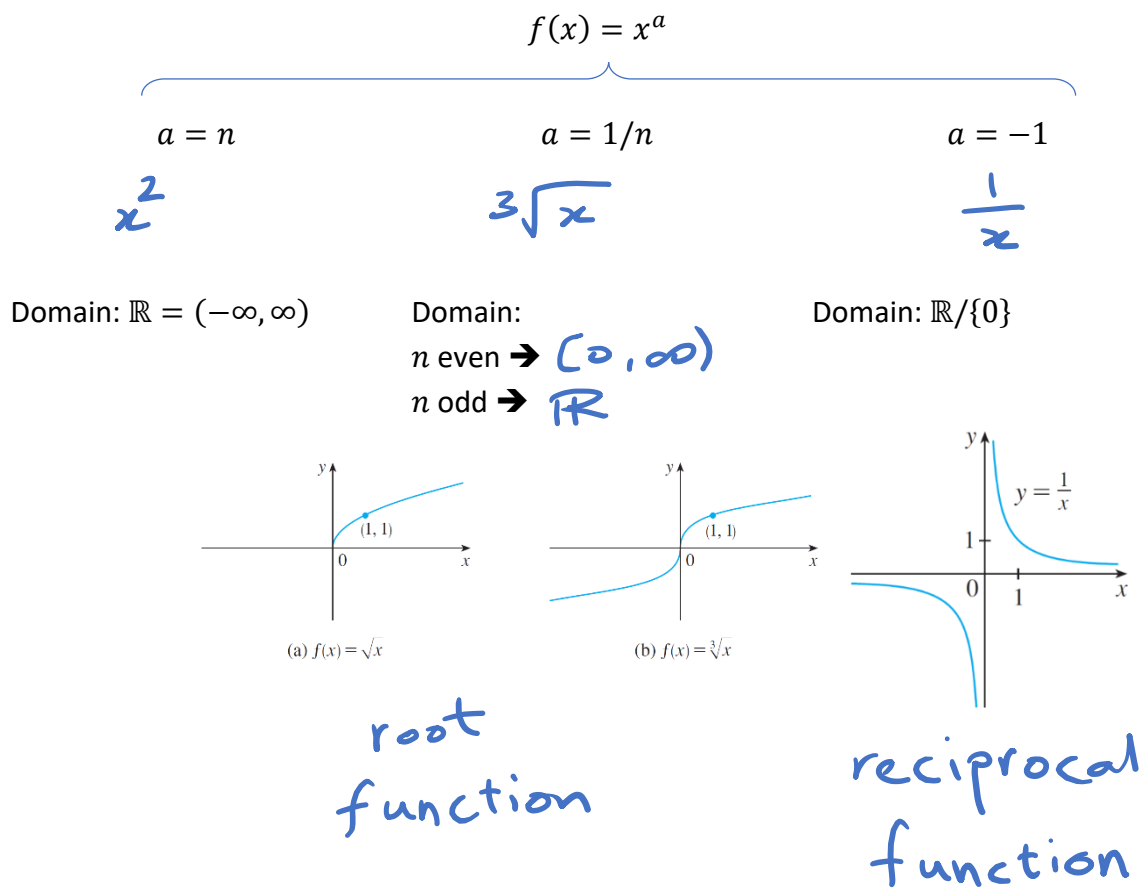
quadratic
formula

تذكر

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

القانون
العام

- Power functions دوال القوى (3/8):



- Rational functions الدوال النسبية (4/8):

$$f(x) = \frac{P(x)}{Q(x)}$$

Domain: $\mathbb{R}/\{\text{zeros of the denominator}\}$

Example 1

Find the domain of the function $f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$

Solution

$$\begin{aligned} x^2 - 4 &= 0 \\ (x-2)(x+2) &= 0 \\ x &= 2 \quad x = -2 \end{aligned}$$

$$\therefore D_f : \mathbb{R} / \{-2, 2\}$$

- Algebraic functions (5/8):

دالة تتكون من تركيب دالتين أو أكثر بالجمع addition، الطرح subtraction، الضرب multiplication، القسمة، والوضع تحت الجذور taking roots

$$f(x) = \sqrt{x^2 + 1} \quad g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$$

Example 2

Find the domain of the function.

(a) $f(x) = \sqrt{x^2 + 1}$

(b) $g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$

Solution

(a) $x^2 + 1 \geq 0$
 x^2 is always +
 $\therefore D_f: \mathbb{R}$

(b) $x + \sqrt{x} \neq 0 \cap x > 0$
 $D_g: (0, \infty)$

تذكر

خصائص الجذور

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

$$\sqrt{x + y} \neq \sqrt{x} + \sqrt{y}$$

$$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

خصائص الضرب والقسمة

$$a(b + c) = ab + ac$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}, \quad c \neq 0$$

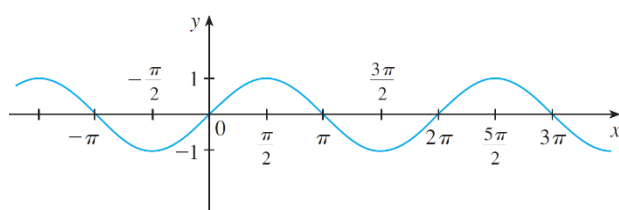
$$\frac{a}{c+b} \neq \frac{a}{c} + \frac{a}{b}$$

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm cb}{bd}$$

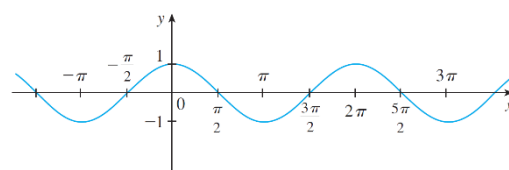
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

- Trigonometric functions الدوال المثلثية (6/8):

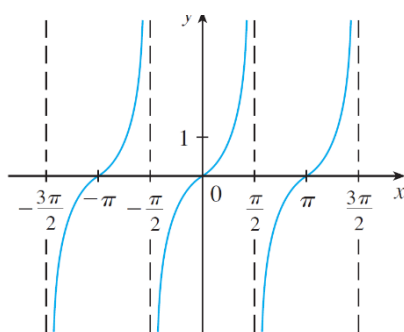
	Domain	Range
$\sin(x)$	$\mathbb{R} = (-\infty, \infty)$	$[-1, 1]$
$\cos(x)$	$\mathbb{R} = (-\infty, \infty)$	$[-1, 1]$
$\tan(x)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$\mathbb{R} = (-\infty, \infty)$



$$y = \sin x$$



$$y = \cos x$$



$$y = \tan x$$

تذكر

$$\sin \theta = \frac{y}{r} = \frac{\text{opp}}{\text{hyp}}$$

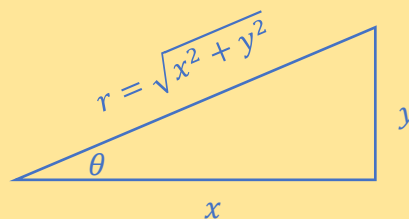
$$\cos \theta = \frac{x}{r} = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{y}{x} = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y}$$



$$\sin(x + 2\pi) = \sin(x), \quad \forall x$$

$$\cos(x + 2\pi) = \cos(x), \quad \forall x$$

$$\tan(x + \pi) = \tan(x), \quad \forall x$$

تذكر

Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

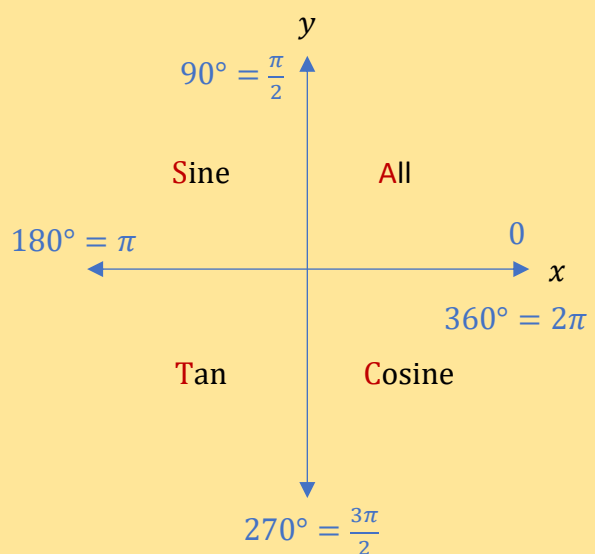
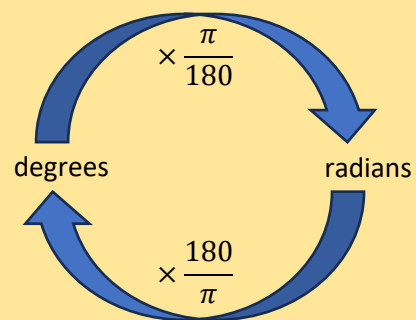
$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - \sin^2 \theta$$

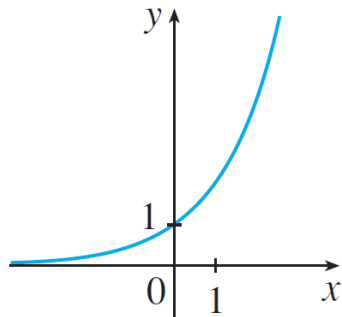
θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
0		0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	DNE
180°	π	0	-1	0
270°	$\frac{3\pi}{2}$	-1	0	DNE
360°	2π	0	1	0



- Exponential functions الدوال الأسية (7/8):

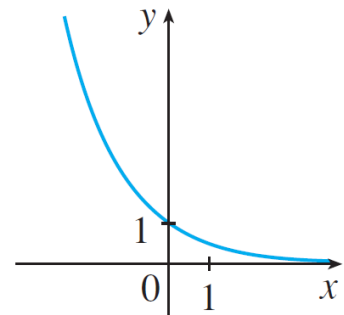
$$f(x) = y = a^x$$

$$a > 1$$



Domain: $\mathbb{R}$
Range: $(0, \infty)$

$$0 < a < 1$$



Domain: $\mathbb{R}$
Range: $(0, \infty)$

- Logarithmic functions الدوال اللوغاريتمية (8/8):

$$f(x) = \log_a x = \frac{\ln x}{\ln a}, a \neq 1$$

$$D_f: (0, \infty)$$

$$R_f: \mathbb{R} = (-\infty, \infty)$$

Example 3

Classify the following functions as one of the types of functions that we have discussed.

(a) $f(x) = 5^x$

(b) $g(x) = x^5$

(c) $h(x) = \frac{1+x}{1-\sqrt{x}}$

(d) $u(t) = 1 - t + 5t^4$

Solution

(a) Exponential

(b) Power

(c) Algebraic

(d) Polynomial, degree = 4

Problems

- Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

(a) $f(x) = \log_2 x$

(b) $g(x) = \sqrt[4]{x}$

(c) $h(x) = \frac{2x^3}{1-x^2}$

(d) $u(t) = 1 - 1.1t + 2.54t^2$

(e) $v(t) = 5^t$

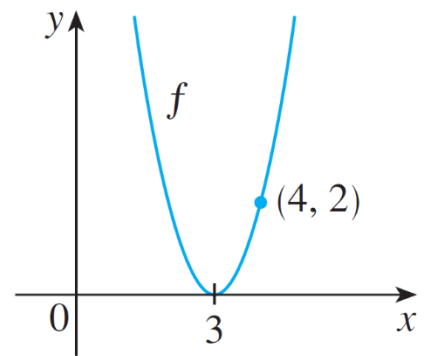
(f) $w(\theta) = \sin \theta \cos^2 \theta$

- Find the domain of the function.

(a) $g(x) = \frac{1}{1 - \tan x}$

- Find expressions for the quadratic functions whose graphs are shown.

(a)



(b)

