

Section 1.4 – Exponential Functions

- The exponential function الدالة الأسية:

$$f(x) = a^x$$

where $a > 0$

لاحظ أن في الدالة الأسية يكون الأساس رقم ثابت و المتغير x يكون في الأس exponent
غير دالة القوى power function يكون المتغير x في الأساس والثابت هو الأس، مثلا x^2

Example 1

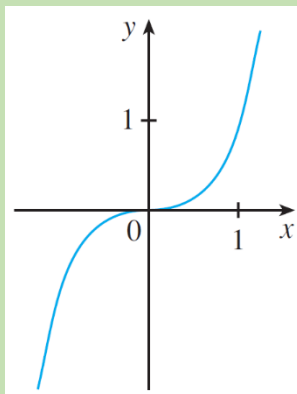
Compare the exponential function 3^x and the power function x^3 .

Solution

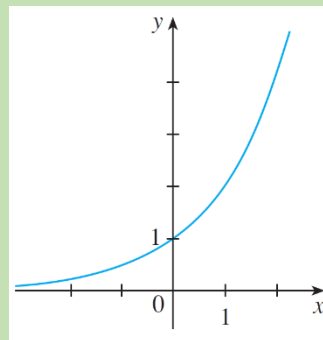
Growth:

x	$f(x) = x^3$	$f(x) = 3^x$
-1	$(-1)^3 = -1$	$3^{-1} = \frac{1}{3}$
0	$0^3 = 0$	$3^0 = 1$
1	$1^3 = 1$	$3^1 = 3$
2	$2^3 = 8$	$3^2 = 9$
4	$4^3 = 64$	$3^4 = 81$
6	$6^3 = 216$	$3^6 = 729$

$f(x) = 3^x$
grows much
more rapidly



Domain: $\mathbb{R}$
Range: $\mathbb{R}$



$D: \mathbb{R}$
 $R: (0, \infty)$

لاحظ

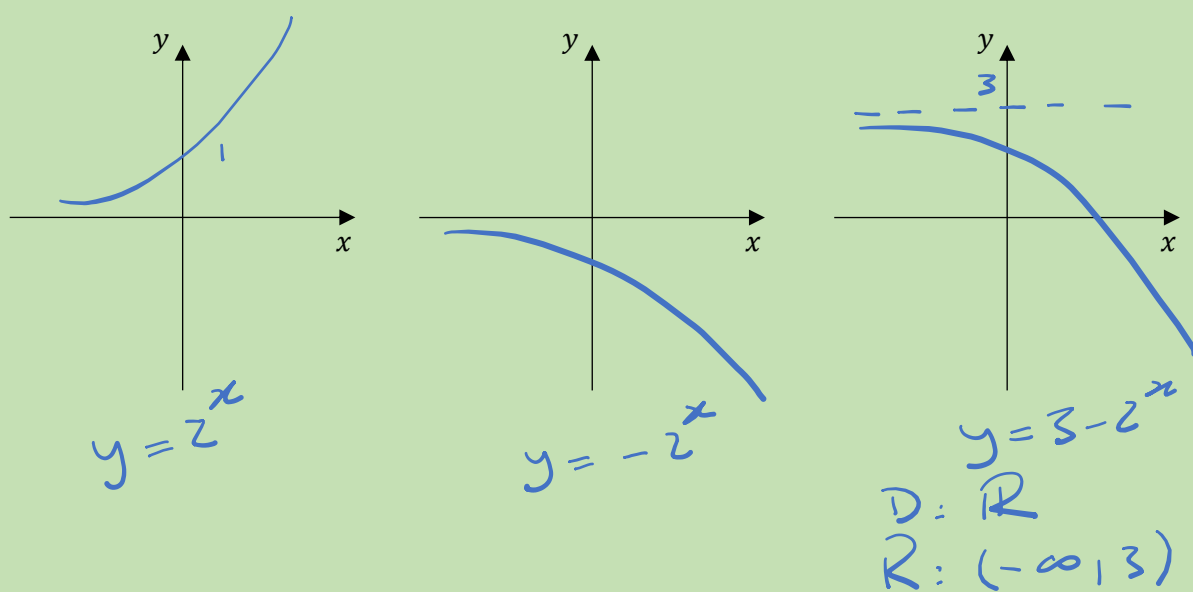
Special values of x for the exponential function $f(x) = a^x$

$x = 0$ $f(x) = a^0 = 1$	$x = -n$ $f(x) = a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$x = p/q$ (ration) $f(x) = a^{p/q} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$
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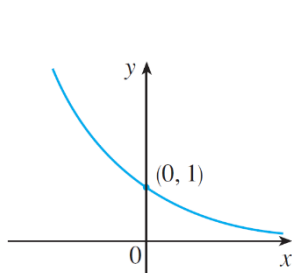
Example 2

Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.

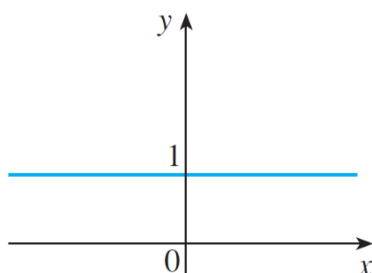
Solution



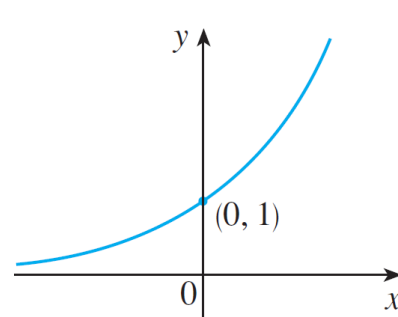
- Growth and decay



$$y = a^x, 0 < a < 1$$



$$y = 1^x$$



$$y = a^x, a > 1$$

تذكر

Laws of exponents

$$1. a^x a^y = a^{x+y}$$

$$2. \frac{a^x}{a^y} = a^{x-y}$$

$$3. (ab)^x = a^x b^x$$

$$4. \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$5. (a^x)^y = a^{xy}$$

$$6. a^\infty = \infty$$

$$7. a^{-\infty} = \frac{1}{a^\infty} = 0$$

Example 3

Use the Law of Exponents to rewrite and simplify the expression $x(3x^2)^3$

Solution

$$\begin{aligned} & x(3^3 x^{2 \cdot 3}) \\ &= x(27 x^6) \\ &= 27 x^7 \end{aligned}$$

Problems

- Use the Law of Exponents to rewrite and simplify the expression.

(a) $8^{4/3}$

$$(\sqrt[3]{8})^4 = 2^4 = 16$$

(d) $\frac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$

$$= x^{2n + 3n - 1 - (n + 2)}$$

$$= x^{5n - 1 - n - 2}$$

$$= x^{4n - 3}$$

(g) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

$$= \frac{(a b^{1/2})^{1/2}}{(a b)^{1/3}}$$

$$= \frac{a^{1/2} b^{1/2 \cdot 1/2}}{a^{1/3} b^{1/3}} = a^{1/2 - 1/3} b^{1/4 - 1/3}$$

$$= a^{1/6} b^{-1/12}$$

$$= \frac{a^{1/6}}{b^{1/12}}$$

- Find the domain of each function.

(a) $g(t) = \sqrt{10^t - 100}$

$$10^t - 100 \geq 0$$

$$10^t \geq 100$$

$$10^t \geq 10^2$$

$$t \geq 2$$

$$D_g: [2, \infty)$$

(b) $g(t) = \sin(e^t - 1)$

$$D_g: \mathbb{R}$$

- Find the exponential function $f(x) = Cb^x$ whose graph is given.

(a)

$$6 = Cb^1$$

$$Cb = 6 \rightarrow \textcircled{1}$$

$$24 = Cb^3$$

$$= Cb \cdot b^2$$

$$\frac{24}{6} = \frac{Cb}{C} b^2$$

$$b^2 = 4 \Rightarrow$$

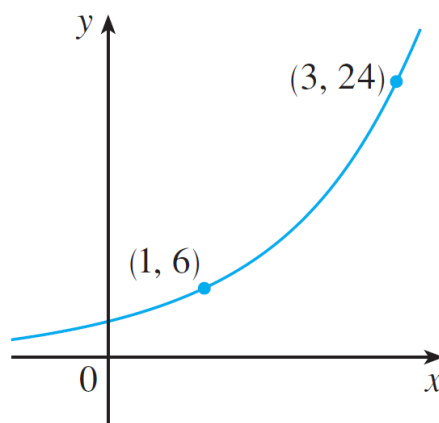
$$\boxed{b = 2}$$

or $b = -2$
~~X rejected~~
 b must be positive

$$C \cdot 2 = 6$$

$$\therefore C = 3$$

$$f(x) = 3 \cdot 2^x$$



(b)

$$3 = Cb^{-1}$$

$$\frac{C}{b} = 3$$

$$C = 3b$$

$$\frac{4}{3} = Cb^1$$

$$\frac{4}{3} = \frac{3b \cdot b}{3}$$

$$\frac{4}{9} = b^2 \Rightarrow$$

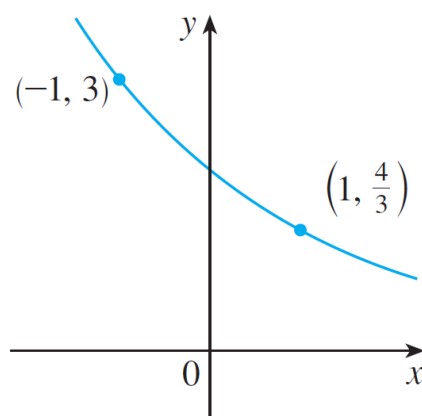
$$\boxed{b = \frac{2}{3}}$$

or $b = -\frac{2}{3}$

~~X rejected~~

$$C = 3 \cdot \frac{2}{3} = 2$$

$$\therefore f(x) = 2 \left(\frac{2}{3}\right)^x$$



- If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right)$$

$$f(x+h) = 5^{x+h}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{5^{x+h} - 5^x}{h} \\ &= \frac{5^x \cdot 5^h - 5^x}{h} \\ &= \frac{5^x (5^h - 1)}{h} \\ &= 5^x \left(\frac{5^h - 1}{h} \right) \end{aligned}$$