Section 1.4 – Exponential Functions

- The exponential function الدالة الأسية:

$$f(x) = a^x$$

where $a > 0$

لاحظ أن في الدالة الأسية يكون الأساس رقم ثابت و المتغير x يكون في الأس exponent

غير دالة القوى power function يكون المتغير x في الأساس والثابت هو الأس، مثلا x^2

Example 1

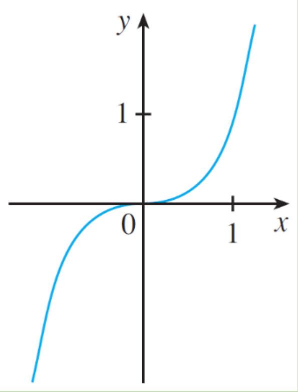
Compare the exponential function 3^x and the power function x^3.

Solution

Growth:

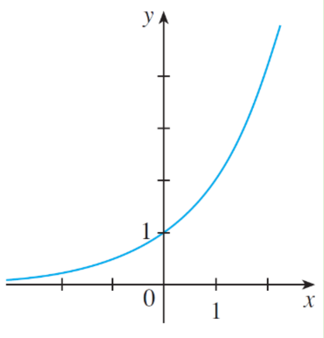
x	$f(x)=x^3$	$f(x)=3^x$
-1	$(-1)^3=-1$	$3^{-1}=\frac{1}{3}$
0	$0^3=0$	$3^0=1$
1	$1^3=1$	$3^1=3$
2	$2^3=8$	$3^2=9$
4	$4^3=64$	$3^4=81$
6	$6^3=216$	$3^6=729$

$f(x)=3^x$ grows much more rapidly



Domain: $\mathbb{R}$

Range: $\mathbb{R}$



D: $\mathbb{R}$

R: $(0,\infty)$

لاحظ

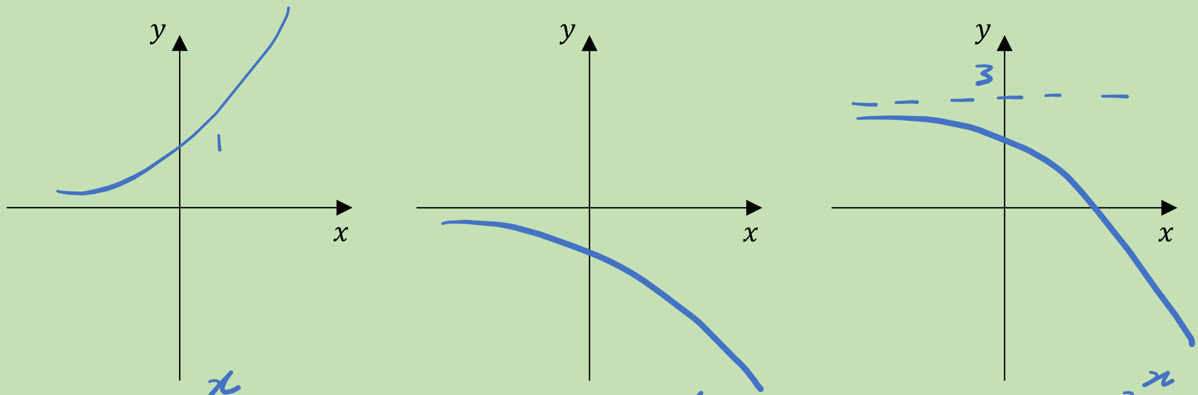
Special values of x for the exponential function $f(x) = a^x$

$x = 0$	$x = -n$	$x = p/q$ (ration)
$f(x) = a^0 = 1$	$f(x) = a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$	$f(x) = a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

Example 2

Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.

Solution

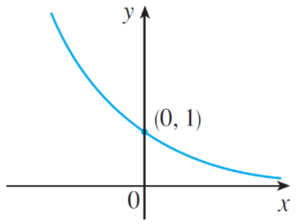


$y = 2^x$ $\qquad$ $y = -2^x$ $\qquad$ $y = 3 - 2^x$

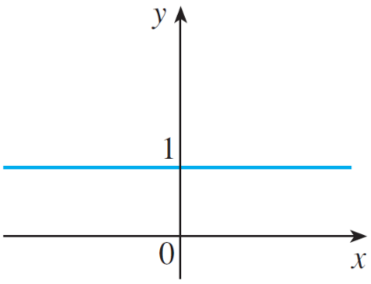
D: $\mathbb{R}$

R: $(-\infty, 3)$

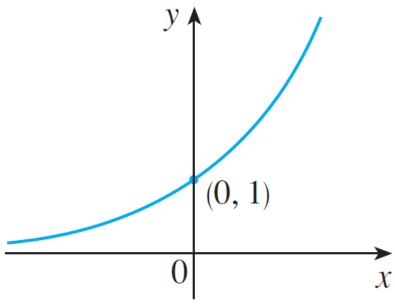
- Growth and decay



$y = a^x,\ 0 < a < 1$



$y = 1^x$



$y = a^x,\ a > 1$

تذكر

Laws of exponents

1. $a^x a^y = a^{x+y}$

2. $\dfrac{a^x}{a^y} = a^{x-y}$

3. $(ab)^x = a^x b^x$

4. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

5. $(a^x)^y = a^{xy}$

6. $a^{\infty} = \infty$

7. $a^{-\infty} = \dfrac{1}{a^{\infty}} = 0$

Example 3

Use the Law of Exponents to rewrite and simplify the expression $x(3x^2)^3$

Solution

$x(3^3 x^{2\cdot 3})$

$= x(27 x^6)$

$= 27x^7$

Problems

- Use the Law of Exponents to rewrite and simplify the expression.

(a) $8^{4/3}$

(d) $\frac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$

(g) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

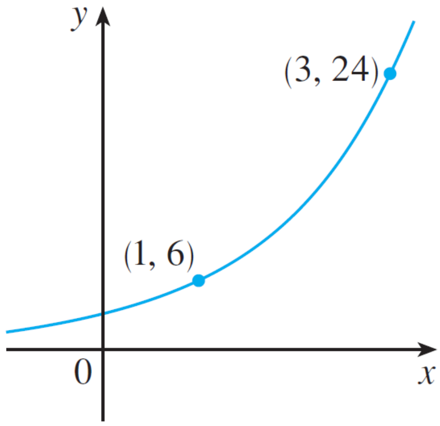
- Find the domain of each function.

(a) $g(t) = \sqrt{10^t - 100}$

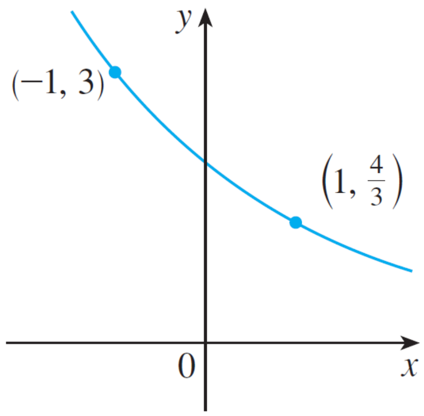
(b) $g(t) = \sin(e^t - 1)$

- Find the exponential function $f(x) = Cb^x$ whose graph is given.

(a)



(b)



- If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x\left(\frac{5^h - 1}{h}\right)$$