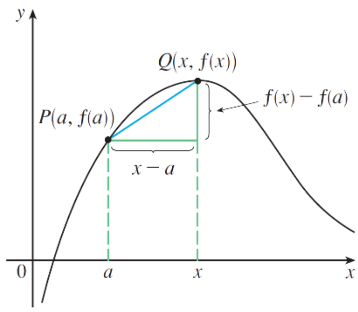
Section 2.7 – Derivatives and Rates of Change

- Slope of the secant line   خط يقطع المنحنى في نقطتين



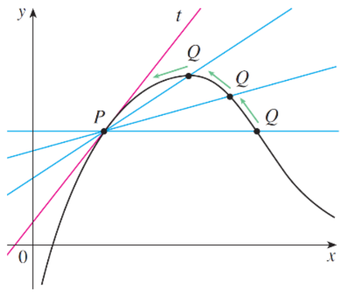
ميل الخط $$m = \frac{f(x) - f(a)}{x - a} = \frac{y - y_0}{x - x_0}$$

- Tangent line: خط يلمس المنحنى عند نقطة

The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that the limit exists.



Example 1

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1,1)$.

Solution

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad a = 1 \ , \ f(a) = 1$$

$$m = \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1}$$

$$= \lim_{x \to 1} x + 1 = 2$$

$$y - y_0 = m(x - x_0)$$

$$y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$$

لاحظ

معادلة المماس

$$y - y_0 = m(x - x_0)$$

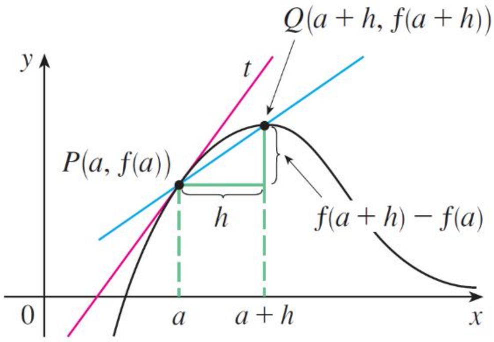
معادلة خط عمودي

$$y - y_0 = -\frac{1}{m}(x - x_0)$$

- h-definition

Take $h = x - a$ $\Rightarrow x = a + h$

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$



Example 2

Find an equation of the tangent line to the hyperbola $y = 3/x$ at the point $(3,1)$.

Solution

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \quad a = 3, \ f(a) = 1$$

$$f(a+h) = \frac{3}{3+h}$$

$$m = \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - \frac{3+h}{3+h}}{h} = \lim_{h \to 0} \frac{\frac{3 - 3 - h}{3+h}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h(3+h)} = -\frac{1}{3}$$

$$y - y_0 = m(x - x_0) \Rightarrow y - 1 = -\frac{1}{3}(x - 3)$$

- Derivative الاشتقاق:

The derivative of a function f at a number a, denoted by $f'(a)$, is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

نفس معادلة ميل الخط

is this limit exists.

Example 3

Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a.

Solution

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(a+h)^2 - 8(a+h) + 9 - (a^2 - 8a + 9)}{h}$$

$$= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - 8a - 8h + 9 - a^2 + 8a - 9}{h}$$

$$= \lim_{h \to 0} \frac{2ah + h^2 - 8h}{h}$$

$$= \lim_{h \to 0} \frac{h(2a + h - 8)}{h}$$

$$= 2a - 8$$

Example 4

Find the equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at the point $(3, -6)$.

Solution

$y - y_0 = m(x - x_0)$, $m = f'(a)$

$m = 2a - 8$

$= 2 \cdot 3 - 8 = -2$

$y - (-6) = -2(x - 3)$

$y + 6 = -2x + 6$

$y = -2x$

لاحظ

$m = f'(a)$ لذن

$y - y_0 = f'(a)(x - x_0)$ معادلة المماس

Problems

- (a) Find the slope of the tangent line to the parabola $y = x - x^3$ at the point $(1, 0)$.

(i) using $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

(ii) using $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

(b) Find an equation of the tangent line in part (a).

- Find an equation of the tangent line to the curve at the given point.

(a) $y = x^3 - 3x + 1, \qquad (2, 3)$

(b) $y = \sqrt{x}, \ (2, \sqrt{2})$

- (a) Find the slope of the tangent to the curve $y = 1/\sqrt{x}$ at the point where $x = a$.

(b) Find equations of the tangent lines at the points $(1, 1)$ and $\left(4, \frac{1}{2}\right)$.

- Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

- If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$, find $f(4)$ and $f'(4)$.

- If $F(x) = 5x/(1 + x^2)$, find $F'(2)$ and use it to find an equation of the tangent line to the curve $y = 5x/(1 + x^2)$ at the point $(2, 2)$.

- If $G(x) = 4x^2 - x^3$, find $G'(a)$ and use it to find equations of the tangent lines to the curve $y = 4x^2 - x^3$ at the points $(2, 8)$ and $(3, 9)$.

- Find $f'(a)$.

(a) $f(t) = 2t^3 + t$

(b) $f(x) = \frac{4}{\sqrt{1-x}}$

- Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

(a) $\lim_{h \to 0} \frac{e^{-2+h} - e^{-2}}{h}$

(b) $\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$