Section 3.1 – Derivatives of Polynomials and Exponential Functions

- Rules of differentiation

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Example 1

Find the derivative for the following functions

(a) $f(x) = x^6$

(b) $y = x^{1000}$

(c) $f(x) = \frac{1}{x^2}$

(d) $\sqrt[3]{x^2}$

Solution

(a) $f'(x) = 6x^5$

(b) $\frac{dy}{dx} = 1000x^{999}$

(c) $f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3} = \frac{-2}{x^3}$

(d) $\sqrt[3]{x^2} = x^{2/3}$

$f' = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

تذكر

ميل خط مستقيم عند نقطة هو نفسه قيمة الاشتقاق عند النقطة

$m = f'(a)$

Example 2

Find equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point $(1,1)$.

Solution

$y = x \cdot x^{1/2} = x^{3/2}$

$y' = \frac{3}{2} x^{1/2}$

$m = y'(1) = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{3}{2}(x-1)$ tangent

$y - 1 = -\frac{2}{3}(x-1)$ normal

- في حالة الضرب في ثابت:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Example 3

Find the derivative of

(a) $y = 3x^4$ (b) $f(x) = -x$

Solution

(a) $y' = 3 \cdot 4x^3 = 12x^3$

(b) $\frac{d}{dx}f(x) = -1$

- في حالة الجمع والطرح:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Example 4

Let $f(x) = x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5$

Solution

$f'(x) = 8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$

Example 5

Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

Solution

$y' = 4x^3 - 12x$

horizontal line $\Rightarrow m = 0$

$4x^3 - 12x = 0$

$4x(x^2 - 3) = 0$

$4x = 0$ or $x^2 - 3 = 0$

$x = 0$ $\quad x = \pm\sqrt{3}$

The curve has horizontal tangent lines at

$x = 0$, $x = \sqrt{3}$, and $x = -\sqrt{3}$

The corresponding points are

$(0, 4)$

$(\sqrt{3}, -5)$

$(-\sqrt{3}, -5)$

- Exponential function rule

$$\frac{d}{dx}a^x = a^x \ln a$$

Example 6

Compute the derivative of the function $f(x) = a^x$ using the definition of the derivative.

Solution

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} \qquad a^{x+h} = a^x \cdot a^h$$

$$= \lim_{h\to 0} \frac{a^x (a^h - 1)}{h}$$

$$= a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$

$$= a^x \ln a$$

$$\lim_{h\to 0} \frac{a^h - 1}{h} = \ln a \qquad \text{li...}$$

Example 7

Differentiate the function $j(x) = x^{24} + e^{24}$

Solution

$j' = 24x^{23}$

لاحظ

الأساس e حالة خاصة كالتالي:

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^{f(x)} = f'(x) \cdot e^{f(x)}$$

Problems

- Differentiate the function

(a) $f(x) = 186.5$

(b) $f(x) = 5.2x + 2.3$

(c) $f(t) = 2t^3 - 3t^2 - 4t$

(d) $g(x) = x^2(1 - 2x)$

(e) $F(r) = \frac{5}{r^3}$

(f) $R(a) = (3a + 1)^2$

(g) $y = 3e^x + \frac{4}{\sqrt[3]{x}}$

(h) $y = \frac{x^2+4x+3}{\sqrt{x}}$

- Find an equation of the tangent line to the curve at the given point

(a) $y = 2e^x + x, \quad (0, 2)$

(b) $y = \sqrt[4]{x} - x, \quad (1, 0)$

- For what value of x does the graph of $f(x) = e^x - 2x$ have a horizontal tangent?

- Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

- Find an equation of the tangent line to the curve $y = x^4 + 1$ that is parallel to the line $32x - y = 15$.

- Find an equation of the normal line to the curve $y = \sqrt{x}$ that is parallel to the line $2x + y = 1$.

- Evaluate $\lim_{x\to 1} \frac{x^{1000}-1}{x-1}$.

- Evaluate $\lim_{x\to 3} \frac{e^{(x-3)^2}-1}{x^2+x-12}$.