Section 3.2 – The Product and Quotient Rules

- The Product Rule قاعدة الضرب

$$\frac{d}{dx}[f(x)\cdot g(x)] = fg' + f'g$$

Example 1

Find the derivative of the function

(a) $f(x) = xe^x$ (b) $g(x) = x^2(1-2x)$

Solution

(a) $f' = xe^x + 1\cdot e^x = xe^x + e^x$

(b) $g' = x^2(-2) + 2x(1-2x)$

$= -2x^2 + 2x - 4x^2$

$= -6x^2 + 2x$

Example 2

If $f(x) = \sqrt{x}g(x)$, where $g(4) = 2$ and $g'(4) = 3$, find $f'(4)$.

Solution

$f'(x) = \sqrt{x}\, g'(x) + \frac{1}{2}x^{-1/2} g(x)$

$f'(4) = \sqrt{4}\, g'(4) + \frac{1}{2\sqrt{4}} g(4)$

$= 2\cdot 3 + \frac{1}{2\cdot 2}\cdot 2$

$= 6 + \frac{1}{2} = \frac{13}{2}$

- The Quotient Rule قاعدة القسمة

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{gf' - fg'}{g^2}$$

Example 3

Let $y = \frac{x^2+x-2}{x^3+6}$, find y'.

Solution

$$y' = \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$= \frac{2x^4 + x^3 + 12x + 6 - 3x^4 - 3x^3 + 6x^2}{(x^3+6)^2}$$

$$= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}$$

Example 4

Find an equation of the tangent line to the curve $y = \frac{e^x}{(1+x^3)}$ at the point $\left(1, \frac{1}{2}e\right)$.

Solution

$$y' = \frac{(1+x^3)e^x - e^x(3x^2)}{(1+x^3)^2}$$

$$m = y'(1) = \frac{2e - 3e}{4} = -\frac{e}{4}$$

$$y - \frac{1}{2}e = -\frac{e}{4}(x-1)$$

الخلاصة

$\frac{d}{dx}(c) = 0$ مشتقة الثابت		$\frac{d}{dx}(x^n) = nx^{n-1}$ قاعدة الأس
$\frac{d}{dx}(a^x) = a^x \ln a$ Exponential function	$\frac{d}{dx}(e^x) = e^x$ base e	$\frac{d}{dx}(e^{f(x)}) = f'(x) \cdot e^{f(x)}$ e أس دالة (أي شيء غير x)
$\frac{d}{dx}(cf) = c \cdot f'$ دالة ضرب ثابت		
$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$ قاعدة الجمع (polynomial)		$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$ قاعدة الطرح (polynomial)
$\frac{d}{dx}[f(x) \cdot g(x)] = fg' + f'g$ ضرب دالتين		$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g - fg'}{g^2}$ قسمة دالتين

Problems

- Differentiate

(a) $g(x) = \left(x + 2\sqrt{x}\right)e^x$

(b) $y = \frac{e^x}{1+x}$

(c) $f(t) = \frac{2t}{4-t^2}$

(d) $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$

(e) $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)$

(f) $y = \frac{\sqrt{x}}{2+x}$

(g) $y = \frac{t^3+3t}{t^2-4t+3}$

(h) $y = \frac{v^3-2v\sqrt{v}}{v}$

(i) $f(x) = \frac{x^2 e^x}{x^2 + e^x}$

- Find $f'(x)$ and $f''(x)$

(a) $f(x) = \sqrt{x}e^x$

(b) $f(x) = \frac{x}{x^2-1}$

- Find equations of the tangent line and normal line to the given curve at the specified point.

$$y = 2xe^x, \quad (0, 0)$$

- If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, find an equation of the tangent line to the graph of g at the point where $x = 3$.