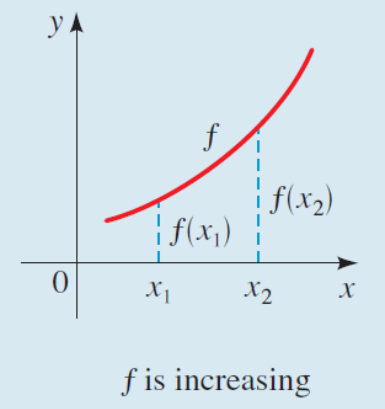
Section 4.1 – Maximum and Minimum Values

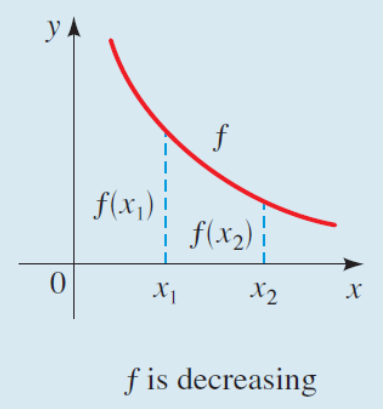
- **Increasing function**

f is increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I

- **Decreasing function**

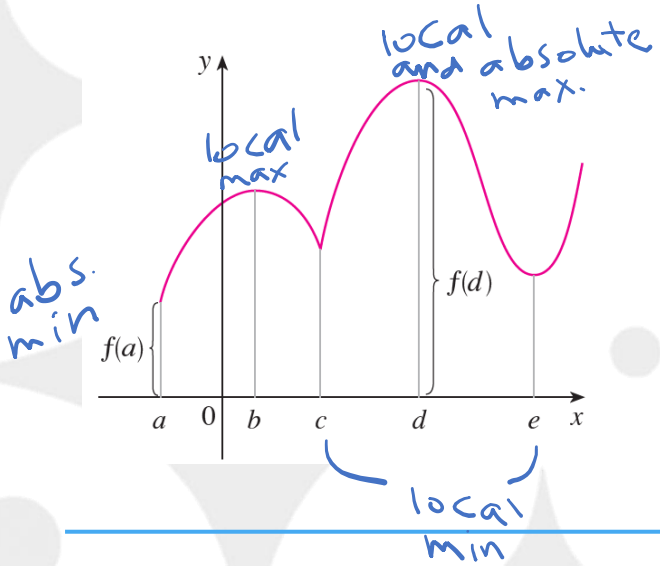
f is decreasing on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I

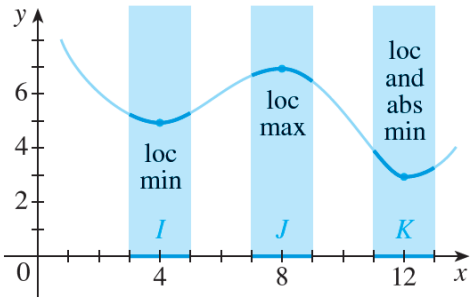




- **Extrema of a function** القيم القصوى

Local		**Absolute**	
Maximum	Minimum	Maximum	Minimum
Let c be a number in the domain D of a function f. Then $f(c)$ is a • local maximum value of f if $f(c) \geq f(x)$ when x is near c.	Let c be a number in the domain D of a function f. Then $f(c)$ is a • local minimum value of f if $f(c) \leq f(x)$ when x is near c.	Let c be a number in the domain D of a function f. Then $f(c)$ is the • absolute maximum value of f on D if $f(c) \geq f(x)$ for all x in D.	Let c be a number in the domain D of a function f. Then $f(c)$ is the • absolute minimum value of f on D if $f(c) \leq f(x)$ for all x in D.



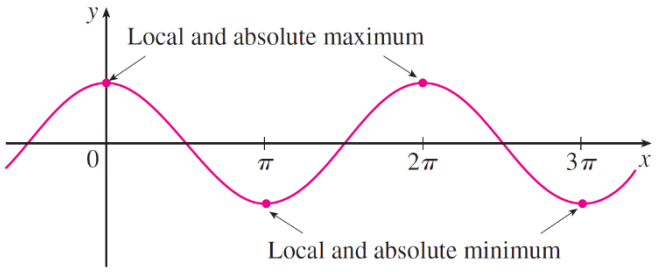


مثال

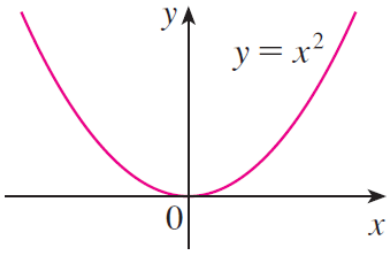
$f(x)=\cos x$ عندها عدد غير نهائي من maximum و minimum

لأن $\cos 2n\pi = 1$ و n تساوي أي عدد صحيح (integer)

و $\cos(2n+1)\pi = -1$



مثال



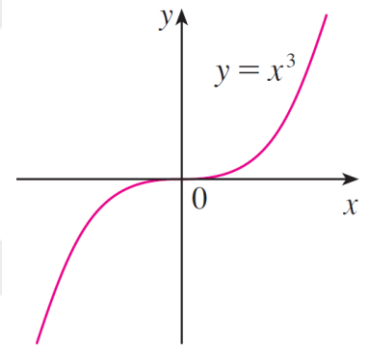
minimum لها $f(x)=x^2$

عند $x=0$ (absolute, local)

لأن $f(x) \geq f(0)$ لكل قيم x

وليس لها max. لأنها تزيد إلى ما لا نهاية

مثال

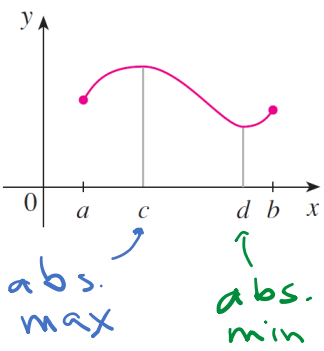


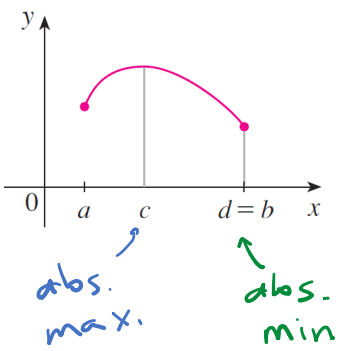
$f(x)=x^3$

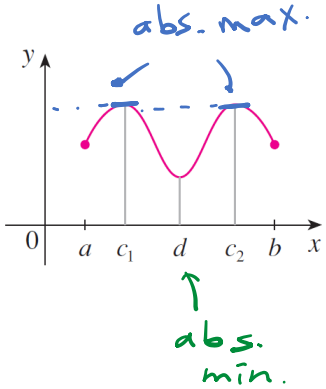
ليس لها extreme values (max. or min.)

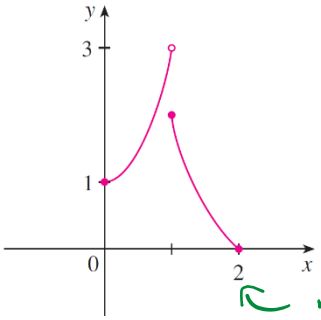
- **The Extreme Value Theorem**:

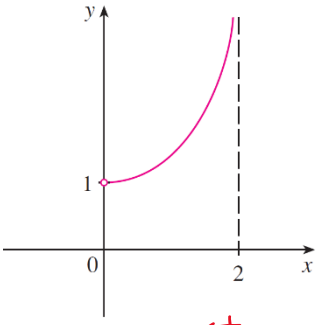
If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.









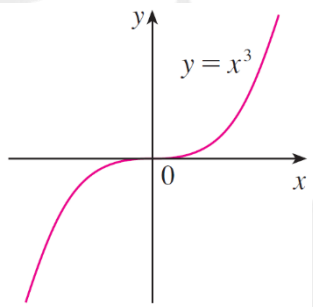


- **Fermat's Theorem:**

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

والعكس غير صحيح

مثال



$f(x) = x^3$

$f'(x) = 3x^2$

$f'(0) = 0$

لكن $x=0$ ليست min أو max

- A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example 1

Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

Solution

$$f'(x) = x^{3/5}(-1) + \frac{3}{5}x^{-2/5}(4-x)$$

$$= -x^{3/5} + \frac{3(4-x)}{5x^{2/5}}$$

$$= \frac{-5x + 3(4-x)}{5x^{2/5}}$$

$$= \frac{-5x + 12 - 3x}{5x^{2/5}}$$

$$= \frac{12 - 8x}{5x^{2/5}}$$

$f'(x) = 0 \Rightarrow 12 - 8x = 0 \Rightarrow x = \frac{-12}{-8} = \frac{3}{2}$

$f'(x)$ Does not exist for $x = 0$

∴ critical numbers are 0 and $\frac{3}{2}$

لاحظ

If f has a local maximum or minimum at c, then c is a critical number of f.

أي قيمة x تعطي local min. أو local max. تسمى critical number

- **The Closed interval Method**

لإيجاد absolute max. و absolute min. للدالة
متصلة في فترة مغلقة $[a,b]$

① نوجد critical numbers في الفترة، وقيم الدالة عندها
② نوجد قيم الدالة عند a و b (نهايات الفترة)
③ أكبر قيمة للدالة من الخطوتين السابقتين هي abs. max.
وأصغر قيمة هي abs. min.

Example 2

Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1 \qquad -\frac{1}{2} \le x \le 4$$

Solution

$$f'(x) = 3x^2 - 6x$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0 \quad \text{or} \quad x = 2$$

$$f(0) = 1$$

$$f(2) = 2^3 - 3 \cdot 2^2 + 1 = -3$$

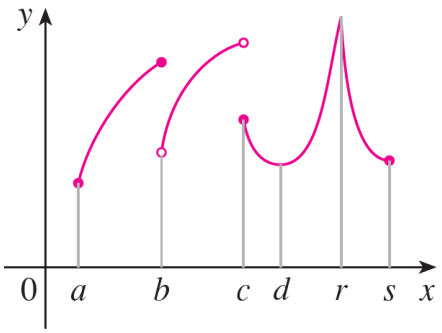
$$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 = \frac{1}{8}$$

$$f(4) = 4^3 - 3 \cdot 4^2 + 1 = 17$$

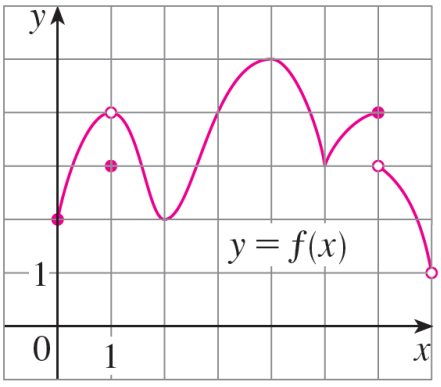
$f(4) = 17$ is abs. max., $f(2) = -3$ is abs. min

Problems

- For each of the numbers a, b, c, d, r, and s, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum.



- Use the graph to state the absolute and local maximum and minimum values of the function.



- Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties: Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4

- (a) Sketch the graph of a function on $[-1, 2]$ that has an absolute maximum but no absolute minimum.

(b) Sketch the graph of a function on $[-1, 2]$ that is discontinuous but has both an absolute maximum and an absolute minimum.

- Find the critical numbers of the function.

(a) $g(x) = \sqrt[3]{4 - x^2}$

(b) $f(\theta) = 2\cos\theta + \sin^2\theta$

(c) $f(x) = x^2e^{-3x}$

- Find the absolute maximum and absolute minimum values of f on the given interval.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 1, \qquad [-2, 3]$

(b) $f(x) = x^{-2} \ln x,$ $\left[\frac{1}{2}, 4\right]$

(c) $f(x) = xe^{x/2}$, $[-3, 1]$