Section 4.7 – Optimization Problems

- Steps In Solving Optimization Problems opt. خطوات حل مسائل

1. Identify

تعرف على
المعطيات
المطاليب
الشروط

2. Draw a Diagram

الرسم إن أمكن

3. Introduce Notation

اكتب رمز لكل شيء وعرّف الكمية المطلوب حلها، مثلاً Q

4. Express Q in terms of some of the other symbols from Step 3.

اكتب Q بدلالة معطى واحد فقط

5. Find the absolute maximum or minimum

أوجد الحل

Closed Interval Method
لو فترة مغلقة

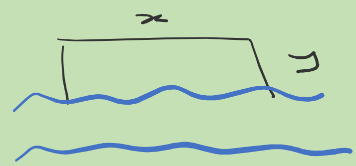
First Derivative Test
لو فترة مفتوحة

Example 1

A farmer has 1200 m of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Solution

$x \geq 0$, $y \geq 0$, $A \geq 0$



$A = xy \rightarrow (1)$

$1200 = 2y + x$

$x = 1200 - 2y \rightarrow (2)$

$A(y) = y(1200 - 2y) = 1200y - 2y^2$

$(1200 - 2y) \geq 0$

$y \geq 0$ $\quad 1200 - 2y \geq 0$

$-2y \geq -1200$

$y \leq 600$

$D = [0, 600]$

$A'(y) = 1200 - 4$

$1200 - 4y = 0 \Rightarrow y = \frac{1200}{4} = 300$

$A(0) = 0$

$A(600) = 600(1200 - 2 \cdot 600) = 0$

$A(300) = 300(1200 - 2 \cdot 300) = 180{,}000 \ m^2$

therefore the max. value is at $y = 300$

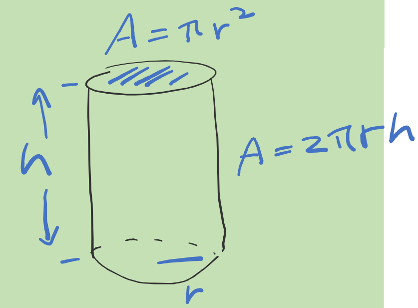
and $x = 1200 - 2(300) = 600 \ m$

by Closed Interval Method

Example 2

Cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

Solution



$r \geq 0$, $h \geq 0$, $A \geq 0$

$A = 2\pi r h + 2\pi r^2 \rightarrow (1)$

$V = 1L = 1000\ cm^3$

$1000 = \pi r^2 h \Rightarrow h = \dfrac{1000}{\pi r^2} \rightarrow (2)$

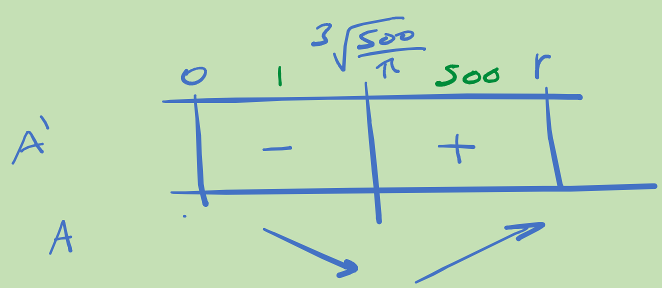
$A(r) = 2\pi r\left(\dfrac{1000}{\pi r^2}\right) + 2\pi r^2$

$= \dfrac{2000}{r} + 2\pi r^2 \qquad D = (0, r)$ ليست مغلقة

$A'(r) = -\dfrac{2000}{r^2} + 4\pi r$

$-\dfrac{2000}{r^2} + 4\pi r = 0 \Rightarrow 4\pi r^3 - 2000 = 0$

$r = \sqrt[3]{\dfrac{2000}{4\pi}} = \sqrt[3]{\dfrac{500}{\pi}}$ critical point



$A(r)$ has a local min at $r = \sqrt[3]{\dfrac{500}{\pi}}$

Since it is the only cp, therefore it is an absolute min.

and $h = \dfrac{1000}{\pi(500/\pi)^{2/3}}$

لاحظ

لا يمكن استخدام CIM في المثال السابق لأن فترة الحل ليست مغلقة، والبديل نستخدم:

First Derivative Test for Absolute Extreme Values

كالتالي:

① نتأكد أن f continuous ، ② لها cp واحدة فقط

If cp is local max ⇒ absolute max.

local min ⇒ absolute min.

Example 3

Find two numbers whose difference is 100 and whose product is a minimum.

Solution

$$a - b = 100 \Rightarrow a = 100 + b$$

$$f(a, b) = ab$$

$$f(b) = b(100 + b) \qquad b = (-\infty, \infty)$$

$$f'(b) = 100 + 2b$$

$$100 + 2b = 0 \Rightarrow b = \frac{-100}{2} = -50$$

	$-\infty$ to -50 (test -100)	-50 to ∞ (test 100)
$f'(b)$	$-$	$+$
$f(b)$	↘	↗

$b = -50$ is a local min.

since it is the only cp then it is an abs. min.

$a = 100 - 50 = 50$

Problems

- The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

- What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

- If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

- Find the point on the line $y = 2x + 3$ that is closest to the origin.

- Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.