

Section 4.9 – Antiderivatives

- Antiderivative

تعريف

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I .

f هي F antiderivative إذا كانت $F'(x) = f(x)$

- If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Example 1

Find the most general antiderivative of each of the following functions.

(a) $f(x) = \sin x$ (b) $f(x) = 1/x$ (c) $f(x) = x^n$, $n \neq -1$

Solution

$$(a) F(x) = -\cos x + C$$

$$\text{because } \frac{d}{dx} (-\cos x) = \sin x$$

$$(b) F(x) = \ln|x| + C$$

$$\text{because } \frac{d}{dx} [\ln|x|] = \begin{cases} \frac{1}{x} & x > 0 \\ -\frac{1}{x} = \frac{1}{x} & x < 0 \end{cases}$$

$$(c) F(x) = \frac{x^{n+1}}{n+1} + C$$

$$\text{because } \frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = \frac{(n+1)x^n}{n+1} = x^n$$

- Table of Antiderivative Formulas

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$\frac{1}{x}$	$\ln x $	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\frac{1}{1+x^2}$	$\tan^{-1} x$
b^x	$\frac{b^x}{\ln b}$	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$

Example 2

Find all functions g such that

$$g'(x) = 4 \sin x + \frac{2x^5 - \sqrt{x}}{x}$$

Solution

$$\begin{aligned}
 g'(x) &= 4 \sin x + \frac{2x^5}{x} - \frac{\sqrt{x}}{x} \\
 &= 4 \sin x + 2x^4 - x^{-1/2} \\
 g(x) &= 4(-\cos x) + \frac{2x^5}{5} - \frac{x^{1/2}}{1/2} + C \\
 &= -4 \cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C
 \end{aligned}$$

Example 3

Find f if $f'(x) = e^x + 20(1+x^2)^{-1}$ and $f(0) = -2$.

Solution

$$f'(x) = e^x + \frac{20}{1+x^2}$$

$$f(x) = e^x + 20 \tan^{-1} x + C$$

$$f(0) = e^0 + 20 \tan^{-1} 0 + C = -2$$

$$C = -2 - 1 = -3$$

$$\therefore f(x) = e^x + 20 \tan^{-1} x - 3$$

Example 4

Find f if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$.

Solution

$$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - 4x + C$$

$$= 4x^3 + 3x^2 - 4x + C$$

$$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + D$$

$$= x^4 + x^3 - 2x^2 + Cx + D$$

$$f(0) = 0 + 0 - 0 + 0 + D = 4 \Rightarrow D = 4$$

$$f(1) = 1 + 1 - 2 + C + 4 = 1 \Rightarrow C + 4 = 1 \Rightarrow C = -3$$

$$\therefore f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

Problems

- Find the most general antiderivative of the function. (Check your answer by differentiation.)

(a) $f(x) = x^2 - 3x + 2$

(b) $f(x) = x(12x + 8)$

(c) $f(x) = e^2$

(d) $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$

(e) $g(t) = \frac{1+t+t^2}{\sqrt{t}}$

- Find f .

(a) $f''(x) = 1/x^2$

(b) $f''(\theta) = \sin \theta + \cos \theta, \quad f(0) = 3, \quad f'(0) = 4$