Section 5.4 – Indefinite Integrals and the Net Change Theorem

- Indefinite integral ≡ Antiderivative notation

$$\int f(x)\,dx = F(x) \qquad \text{means} \qquad F'(x) = f(x)$$

$\int x^2\,dx = \frac{x^3}{3} + C$ because $\frac{d}{dx}\left(\frac{x^3}{3} + C\right) = \frac{3x^2}{3} = x^2$

مهم

$\int cf(x)\,dx = c\int f(x)\,dx$	$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
$\int k\,dx = kx + C$	
$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq 1)$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\int e^x\,dx = e^x + C$	$\int b^x\,dx = \frac{b^x}{\ln b} + C$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$
$\int \sinh x\,dx = \cosh x + C$	$\int \cosh x\,dx = \sinh x + C$

Example 1

Find the general indefinite integral

$$\int (10x^4 - 2\sec^2 x)\,dx$$

Solution

$$= \int 10x^4\,dx - \int 2\sec^2 x\,dx$$

$$= 10\int x^4\,dx - 2\int \sec^2 x\,dx$$

$$= 10\frac{x^5}{5} - 2\tan x + C$$

$$= 2x^5 - 2\tan x + C$$

Example 2

Evaluate

$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

Solution

$$= \int \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}\,d\theta$$

$$= \int \csc\theta \cdot \cot\theta\,d\theta$$

$$= -\csc\theta + C$$

Example 3

Evaluate

$$\int_0^3 (x^3 - 6x)\,dx$$

Solution

$$= \frac{x^4}{4} - \frac{6x^2}{2} \Big|_0^3$$

$$= \left[\frac{3^4}{4} - 3(3)^2\right] - \left[\frac{0}{4} - \frac{0}{2}\right]$$

$$= \frac{81}{4} - 27 = \frac{-27}{4}$$

Example 4

Find

$$\int_0^2 \left(2x^3 - 6x + \frac{3}{x^2+1}\right) dx$$

Solution

$$= \frac{2x^4}{4} - \frac{6x^2}{2} + 3\tan^{-1}x \Big|_0^2$$

$$= \frac{2^4}{2} - 3(2)^2 + 3\tan^{-1}2 - 0$$

$$= 8 - 12 + 3\tan^{-1}2 = -4 + 3\tan^{-1}2$$

Example 5

Find

$$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\,dt$$

Solution

$$\int_1^9 \frac{2t^2}{t^2} + \frac{t^2\sqrt{t}}{t^2} - \frac{1}{t^2}\,dt$$

$$= \int_1^9 2 + \sqrt{t} - t^{-2}\,dt$$

$$= 2t + \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1}\Big|_1^9$$

$$= \left[2\cdot 9 + \frac{2}{3}(9)^{3/2} + \frac{1}{9}\right] - \left[2 + \frac{2}{3} + 1\right]$$

$$= \left[18 + \frac{2}{3}\cdot 27 + \frac{1}{9}\right] - \left[\frac{9}{3} + \frac{2}{3}\right]$$

$$= \left[\frac{324}{9} + \frac{1}{9}\right] - \left[\frac{11}{3}\right]$$

$$= \frac{325}{9} - \frac{11}{3}$$

$$= \frac{325}{9} - \frac{33}{9} = \frac{292}{9}$$

Problems

- Verify by differentiation that the formula is correct.

$$\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

- Find the general indefinite integral.

(a) $\int \sqrt[4]{x^5}\, dx$

(b) $\int \left(u^6 - 2u^5 - u^3 + \frac{2}{7}\right) du$

(c) $\int v(v^2 + 2)^2\, dv$

(d) $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx$

(e) $\int \left(\frac{1+r}{r}\right)^2 dr$

- Evaluate the integral.

(a) $\int_0^3 (1 + 6w^2 - 10w^4)\, dw$

(b) $\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3}\right) dx$

(c) $\int_0^1 (5x - 5^x)\, dx$

(d) $\int_0^2 |2x-1|\, dx$

(e) $\int_0^{3\pi/2} |\sin x|\, dx$