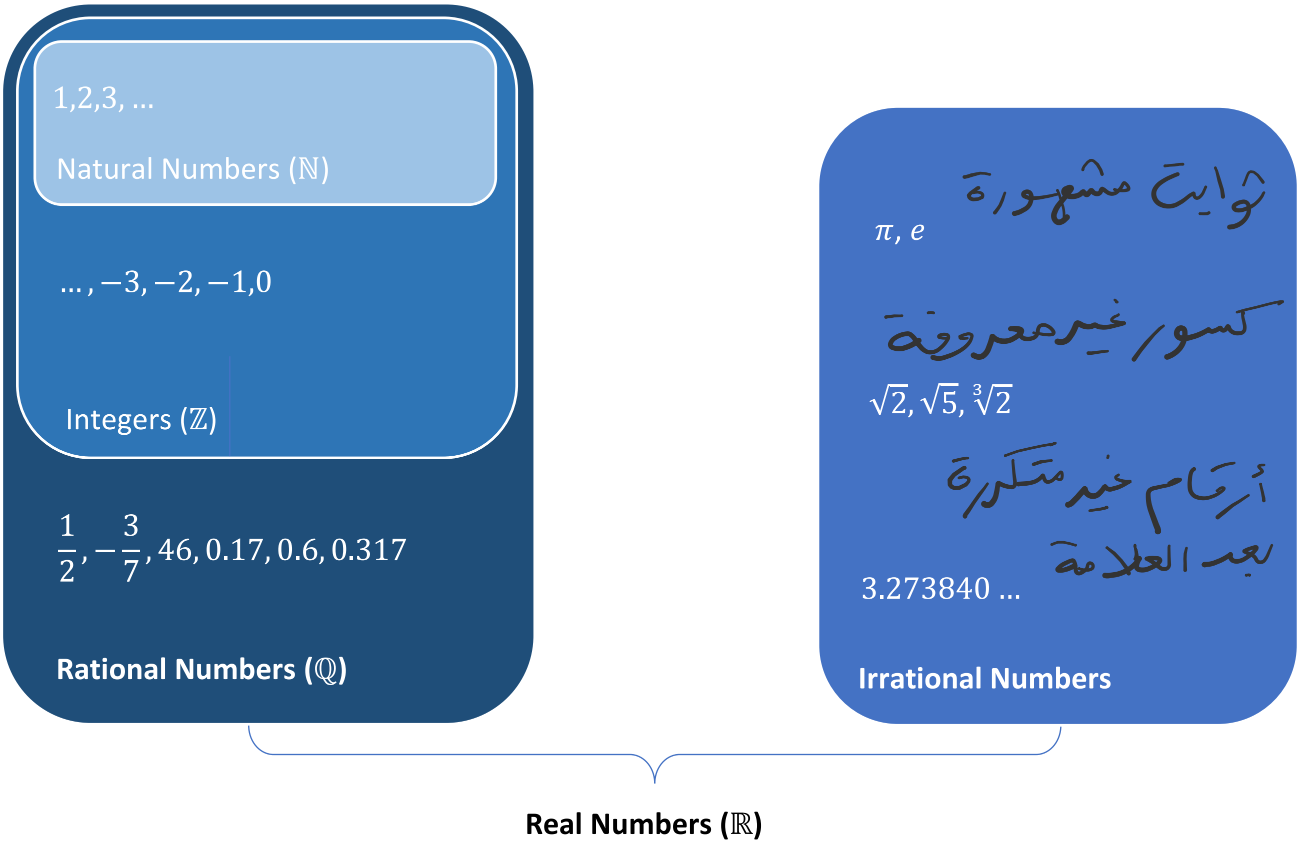
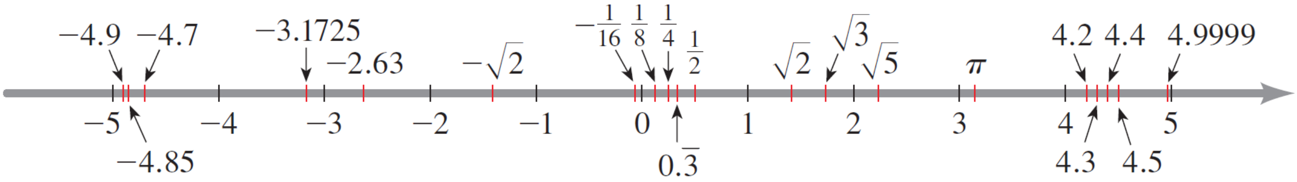
Section 1.1 – Real Numbers





لاحظ

الأعداد النسبية $\mathbb{Q}$ نقدر نعبر عنها بثلاث طرق:

- **Fraction كسر**

$\frac{1}{2}$ $\quad -\frac{3}{7}$ $\quad 46 = \frac{46}{1}$

- **Finite decimal numbers أرقام محدودة بعد العلامة العشرية**

0.17 $\quad 0.5$

- **Repeating decimal representation أرقام متكررة بعد العلامة العشرية**

$0.\overline{6} = 0.66666\ldots$ $\quad 0.3\overline{17} = 0.3171717\ldots$

Example 1

Express the decimal $0.2\overline{8}$ as a fraction.

Solution

$$x = 0.2\overline{8} = 0.28888\ldots$$

$$100x = 28.8888\ldots$$
$$10x = 2.8888\ldots$$

$$\frac{\cancel{90}x}{\cancel{90}} = \frac{26}{90}$$

$$\boxed{x = \frac{26}{90}}$$

- Properties of real numbers خصائص الأعداد الحقيقية

Commutative

$a + b = b + a$ $\quad$ $3 + 4 = 4 + 3$

$ab = ba$ $\quad$ $5 \cdot 6 = 6 \cdot 5$

Associative

$(a + b) + c = a + (b + c)$ $\quad$ $(2 + 3) + 7 = 2 + (3 + 7)$

$(ab)c = a(bc)$ $\quad$ $(4 \cdot 5) \cdot 3 = 4 \cdot (5 \cdot 3)$

Distributive

$a(b + c) = ab + ac$ $\quad$ $2 \cdot (6 + 4) = 2 \cdot 6 + 2 \cdot 4$

$(b + c)a = ab + ac$ $\quad$ $(6 + 4) \cdot 2 = 2 \cdot 6 + 2 \cdot 4$

Example 2

Use properties of real numbers to write the expression without parentheses:

(a) $3(x + y)$

(b) $(3a)(b + c - 2d)$

Solution

(a) $3(x+y) = 3x + 3y$

(b) $(3a)(b+c-2d) = 3ab + 3ac - 6ad$

- The number 0 is called the **additive identity** المحايد الجمعي because $a + 0 = a$ for any real number a.

- **Properties of negatives**

1. $(-1)a = -a$

2. $-(-a) = a$

3. $(-a)b = a(-b) = -(ab)$

4. $(-a)(-b) = ab$

5. $-(a + b) = -a - b$

6. $-(a - b) = b - a$

Example 3

Use properties of negatives to write the expression $-(x + y - z)$ without parentheses.

Solution

$-x - y + z$

- The number 1 is called the **multiplicative identity** المحايد الضربي because $a \cdot 1 = a$ for any real number a.

- Division القسمة is the inverse of multiplication, hence:

$$a \div b = a \cdot \frac{1}{b}$$

$$= \frac{a}{b}$$

Numerator البسط

Denominator المقام

- **Properties of fractions**

1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ $\quad \frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$

2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ $\quad \frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$

3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ $\quad \frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$

4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ $\quad \frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7 + 3 \cdot 5}{5 \cdot 7} = \frac{14+15}{35} = \frac{29}{35}$

5. $\frac{ac}{bc} = \frac{a}{b}$ $\quad \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$

6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ $\quad \frac{2}{3} = \frac{6}{9}$, so $2 \cdot 9 = 3 \cdot 6$

Example 4

Perform the indicated operations.

(a) $\frac{3}{10}+\frac{4}{15}$

(b) $\frac{2}{3}(6-\frac{3}{2})$

(c) $\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{15}}$

Solution

(a) $\frac{3}{10}+\frac{4}{15}$

$$= \frac{3\times15+4\times10}{10\times15} = \frac{45+40}{150}$$

$$= \frac{85}{150}$$

(b) $\frac{2}{3}\left(6-\frac{3}{2}\right) = \frac{2}{3}\cdot 6 - \frac{2}{3}\cdot\frac{3}{2}$

$$= \frac{12}{3} - \frac{6}{6} = 4-1=3$$

(c) $\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{15}} = \left(\frac{2}{5}+\frac{1}{2}\right)\div\left(\frac{1}{10}+\frac{3}{15}\right)$

$$= \left(\frac{2\times2+1\times5}{5\times2}\right)\div\left(\frac{1\times15+3\times10}{10\times15}\right)$$

$$= \left(\frac{9}{10}\right)\div\left(\frac{45}{150}\right)$$

$$= \frac{9}{10}\times\frac{150}{45} = \frac{1350}{450} = 3$$

- If a is a real number, then the **absolute value** مطلق of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

- **Properties of absolute values**

1. $|a| \geq 0$ $\quad$ $|-3| = 3 \geq 0$
2. $|a| = |-a|$ $\quad$ $|5| = |-5|$
3. $|ab| = |a||b|$ $\quad$ $|-3 \cdot 5| = |-3||5|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $\quad$ $\left|\frac{12}{-3}\right| = \frac{|12|}{|-3|}$
5. $|a + b| \leq |a| + |b|$ $\quad$ $|-3 + 5| \leq |-3| + |5|$

Example 5

Evaluate each expression.

(a) $||-6| - |-4||$

(b) $\left|\frac{7-12}{12-7}\right|$

Solution

(a) $|6 - 4| = |2| = 2$

(b) $\left|\frac{-5}{5}\right| = |-1| = 1$

- The distance between any two points a and b on the real line is

$$d(a, b) = |b - a| = |a - b|$$

Example 6

Find the distance between -8 and 2

Solution

$D = |-8 - 2| = |-10| = 10$

- A set المجموعة is a collection of objects called elements العناصر

$A = \{1,2,3,4,5,6\}$

لاحظ الأقواس المستخدمة { }

$A = \{x | x \text{ is an integer and } 0 < x < 7\}$

طريقة أخرى لكتابة

المجموعات تسمى

set-builder notation

- If A and B are sets, then their **union** اتحاد $A \cup B$ is the set that consists of all the elements that are in A and B

Example:

$A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7\}$

$$A \cup B = \{1,2,3,4,5,6,7\}$$

- The **intersection** التقاطع of A and B is the set $A \cap B$ consisting of all elements that are in both A and B.

Example:

$A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7\}$

$$A \cap B = \{4,5\}$$

- إذا كانت المجموعة ليس بها عناصر تسمى مجموعة خالية **empty set** و يرمز لها بالرمز $\emptyset$ (Phi)

Example:

$A = \{1,2,3\}$ and $B = \{4,5,6,7\}$

$$A \cap B = \emptyset$$

- **Intervals** الفترات are sets with an infinite number of elements and are defined by start and end values.

Open interval

$(a, b) = \{x | a < x < b\}$

Closed interval

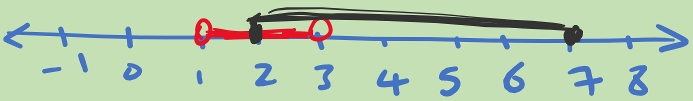
$[a, b] = \{x | a \le x \le b\}$

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \le x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

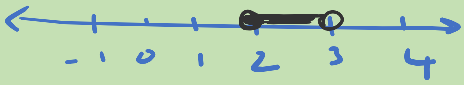
Example 7

Graph each set.
(a) $(1,3) \cap [2,7]$
(b) $(1,3) \cup [2,7]$

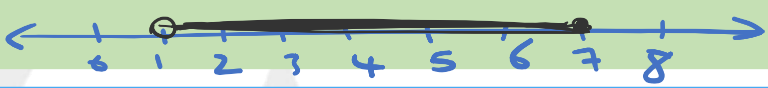
Solution



(a)



(b)



Problems

- Perform the indicated operations.

(a) $\frac{2}{\frac{2}{3}} - \frac{\frac{2}{3}}{2}$

(b) $1 + \frac{5}{8} - \frac{1}{6}$

(c) $(3 + \frac{1}{4})(1 - \frac{4}{5})$

- Find the indicated set if

$A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6, 8\}$, and $C = \{7, 8, 9, 10\}$

(a) $A \cup B \cup C$

(b) $A \cap B \cap C$

- Find the indicated set if

$A = \{x | x \geq -2\}$, $B = \{x | x < 4\}$, and $C = \{x | -1 < x \leq 5\}$

(a) $B \cup C$

(b) $A \cap C$

- Express the inequality in interval notation, and then graph the corresponding interval.

(a) $x \leq 1$

(b) $-2 < x \leq 1$

- Find the distance between the given numbers.

(a) -3 **and** 21

(b) $\frac{11}{8}$ **and** $-\frac{3}{10}$

- Express each repeating decimal as a fraction

(a) $0.2\overline{8}$

(b) $5.\overline{23}$

(c) $2.1\overline{35}$